Math 423/823 Exercise Set 7

Due Thursday, Mar. 15

25. [BC#4.46.10, p.134] Find the integral $\int_C z^n(\overline{z})^m \, dz$, where C is the unit circle $|z| = 1$ traversed in a counterclockwise direction.

    [Note: you will fnd it helpful to know that $\int_0^{2\pi} e^{ik\theta} \, d\theta$ is 0 if $k \neq 0$, and 2π if $k = 0$. You need not show this.]

26. Show that 'integration by parts' works with analytic functions: for any curve $\gamma(t)$, $a \leq t \leq b$, if f, g, f' and g' are all analytic along γ, then we have

$$\int_\gamma f(z)g'(z) \, dz = [f(\gamma(b))g(\gamma(b)) - f(\gamma(a))g(\gamma(a))] - \int_\gamma f'(z)g(z) \, dz$$

    [Hint: $F(z) = f(z)g(z)$ is the antiderivative of what (analytic) function?]

    [Note: we will shortly be learning that the analyticity of f' and g' follow from that of f and g, so the requirements on the derivatives are not, in the end, really neccesary...]

27. (Via the fundamental theorem of ('complex') calculus,) Compute

$$\int_\gamma ze^{iz} \, dz \ ,$$

    where γ is the (unit) circular arc running from $z = 1$ to $z = i$.

    [Hint! Problem #25 will help...]

28. [BC #4.49.4, p.147] Find the integral $\int_C z^{1/2} \, dz$, where

    $f(z) = z^{1/2} = e^{(1/2)\log(z)}$ is the <u>branch</u> of $z^{1/2}$ that uses values of $\log(z)$ with imaginary parts lying between 0 and 2π,

    and C is any curve running from $P = -3$ to $Q = 3$ which (except for its endpoints) lies <u>below</u> the x-axis (i.e., real axis).

    [Hint: Find an antiderivative of f, but make sure you pick a good branch of the function! Example 4 on p.143-4 describes a similar situation.]