Math 445 Homework 5

Due Friday, October 18

20. Find the continued fraction expansions of the rational numbers

$$23/38 \qquad \text{and} \qquad 112/55$$

21. Show that if $a, b \in \mathbb{Z}$, $b \geq 1$, and $(a, b) = 1$, then the continued fraction expansion of a/b has length at most b .

    Hint: this is really a question about the Euclidean algorithm....

22. Find the continued fraction expansion of $\sqrt{15}$, and use this to find the first six (6) convergents of $\sqrt{15}$.

23. Repeat problem # 22, for $\sqrt{23}$.

24. Show that for n a positive integer that is not a perfect square (translation: the continued fraction expansion of $\sqrt{n}$ never terminates), that at every stage of the continued fraction expansion of $x = \sqrt{n}$

$$x = \langle a_0, a_1, \ldots , a_{k-1}, a_k + x_k \rangle$$

x_k is always of the form $x_k = \dfrac{\sqrt{n} - a}{b}$, where $b | n - a^2$. Conclude that the continued fraction expansion of $\sqrt{n}$ will eventually repeat, with a period of length at most $n\lfloor\sqrt{n}\rfloor$.

Hint: by induction! In the inductive step, write $\dfrac{b}{\sqrt{n} - a} = \dfrac{\sqrt{n} + a}{c}$, and then find the fractional part of this. For the second half, how long must you wait before the x_k's *must* repeat themselves?