

## Math 445 Homework 2

Due Wednesday, Sept. 17

5. Show, by induction, that for every  $n \in \mathbb{N}$ ,  $f(n) = \frac{1}{2}n^4 + \frac{1}{3}n^3 + \frac{1}{6}n$  is an integer.  
(Note, however, that it is *not* a multiple of  $n$  !)
6. Show that  $8321=53 \times 157$  is a strong pseudoprime to the base 2.  
[Do the calculations by hand....]
7. Show that  $\gcd(ab, n)$  divides  $[\gcd(a, n)][\gcd(b, n)]$  .  
(There are at least 3 distinct proofs, depending on how you characterize gcd's?)
8. (NZM, Problem 2.4.9) [For a pseudoprime, failing the Miller-Rabin test finds proper factors.]  
Show that if  $x^2 \equiv 1 \pmod{n}$  and  $x \not\equiv \pm 1 \pmod{n}$ , then  $1 < (x - 1, n) < n$  and  $1 < (x + 1, n) < n$  .