Math 445 Homework 5

Due Friday, October 10

17. Show that if $a^2 + 2b^2 = c^2$ and $p|(a,c)$ for some prime p, then $p|b$.
    (Note: $p = 2$ is treated differently...)

18. Show that if $a^2 + 2b^2 = c^2$ and $(a,c) = 1$, then a and c are odd, and b is even.

19. Continuing the analysis: with $(a,c) = 1$, writing
    $$2b^2 = c^2 - a^2 = (c-a)(c+a) = (2u)(2v) ,$$
    show that $(u,v) = 1$, one of u,v is odd and the other even, and $u = 2x^2, v = y^2$ or $u = y^2, v = 2x^2$ for some integers x,y.

20. Finishing the analysis: show that $(2x^2 - y^2)^2 + 2(2xy)^2 = (2x^2 + y^2)^2$, and every solution to $a^2 + 2b^2 = c^2$ with $a,b,c \in \mathbb{N}$ has this form:
    $$a = |2x^2 - y^2|,\ b = 2xy,\ c = 2x^2 + y^2, \text{ with } x,y \geq 0 \text{ and } y \text{ odd} .$$