Math 871 Problem Set 1

Starred (*) problems are due Thursday, Sept. 4.

Note that $\mathbb{Z}_+ = \{n \in \mathbb{Z} \ : \ n \geq 1\}$ (which was not what your instructor thought it was).

1. [Munkres, p.14, #1 (part)] For each statement below, determine whether or not it is true. If true, show why; if not, give an example demonstrating this.

   (a) For any sets A, B, $A \setminus (A \setminus B) = B$.

   (b) For any sets A, B, C, $\{A \subseteq B$ and $A \subseteq C\}$ if and only if $A \subseteq B \cap C$.

   (c) For any sets A, B, C, $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$.

   (*) (d) For any sets A, B, C, D, $\{A \subseteq C$ and $B \subseteq D\}$ implies that $A \times B \subseteq C \times D$.

   (*) (e) For any sets A, B, C, D, $A \times B \subseteq C \times D$ implies that $\{A \subseteq C$ and $B \subseteq D\}$.

   (f) For any sets A, B, C, D, $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

2. [Munkres, p.20, #1] Show that if $f : A \to B$ is a function, then

   (a) If $A_0 \subseteq A$, then $A_0 \subseteq f^{-1}(f(A_0))$; the sets are equal, if f is injective.

   (b) If $B_0 \subseteq B$, then $f(f^{-1}(B_0)) \subseteq B_0$; the sets are equal, if f is surjective.

3. [Munkres, p.20, #2 (part)] If $f : A \to B$ is a function, $A_0, A_1 \subseteq A$, and $B_0, B_1 \subseteq B$, then

   (*) (a) $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1)$

   (b) $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$

   (c) $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$, but equality does not always hold.

   (d) $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$.

4. [Munkres, p.44, #7] If A and B are finite sets, show that the set $B^A = \{f : A \to B\}$ of all functions from A to B is also finite.

5. [Munlres, p.51, #5 (part)] For each of the following sets, determine whether or not it is countable:

   (a) $A = \{f : \{0, 1\} \to \mathbb{Z}\}$, all functions from $0, 1$ to $\mathbb{Z}$

   (*) (b) $B = \{f : \mathbb{Z}_+ \to \mathbb{Z}_+ \ : \ f(n) = 1$ for all $n \geq 3\}$

   (c) $F = \{f : \mathbb{Z}_+ \to \mathbb{Z}_+ \ : \$ there is $N \in \mathbb{Z}_+$ with $f(n) = 1$ for all $n \geq N\}$, all eventually-1 functions.

   (d) $H = \{f : \mathbb{Z}_+ \to \mathbb{Z}_+ \ : \$ there is $N \in \mathbb{Z}_+$ with $f(n) = f(N)$ for all $n \geq N\}$, all eventually constant functions.

   (e) $P = \{f : \mathbb{Z}_+ \to \mathbb{Z}_+ \ : \ n > m$ implies $f(n) > f(m)\}$, all increasing functions.