Math 871 Problem Set 5

Starred (*) problems are due Thursday, October 2.

28. [Munkres, p.101, #1] Show that if $A, B, A_\alpha \subseteq X$, where $(X, \mathcal{T})$ is a topological space, then

(a) If $A \subseteq B$ then $\overline{A} \subseteq \overline{B}$

(*) (b) $\overline{A} \cup \overline{B} = \overline{A \cup B}$

(*) (c) $\cup_\alpha \overline{A_\alpha} \subseteq \overline{\cup_\alpha A_\alpha}$; in general, equality does not hold.

(*) 29. [Munkres, p.101, #8(b), sort of] If $A, B \subseteq X$, where $(X, \mathcal{T})$ is a topological space, then what is the relationship between $\overline{A} \cap \overline{B}$ and $\overline{A \cap B}$? What if one of the sets A, B is closed in X ?

30. Show that if $A \subseteq X$ and X has two topologies $\mathcal{T} \subseteq \mathcal{T}'$, then if $x \in X$ is a limit point of A w.r.t. $\mathcal{T}'$, then it is also a limit point of A w.r.t. $\mathcal{T}$.

31. We showed in class that if $A_i \subseteq X_i$ for all $i \in I$, then

$$\overline{\prod_i A_i} = \prod_i \overline{A_i} \subseteq \prod_i X_i$$

when we put the product topology on $\prod_i X_i$. Show that the same is also true if we instead use the box topology on $\prod_i X_i$.

32. Find the closure of the set $A = \{1 - \frac{1}{n} \ : \ n \in \mathbb{Z}_+\} \subseteq \mathbb{R}$, when $\mathbb{R}$ has the

(a) finite complement topology

(b) infinite (open) ray to the right topology

(c) discrete topology

(d) *lower limit topology*, generated by the basis $\mathcal{B} = \{[a, b) : a, b \in \mathbb{R}\}$

(e) countable complement topology.

(*) 33. (a) If $f, g : (X, \mathcal{T}) \to (\mathbb{R}, \text{usual})$ are both continuous, show that $\{x \in X \ : \ f(x) \geq g(x)\}$ is a closed subset of X. [Note that the other set, $\{x \in X \ : \ g(x) \geq f(x)\}$, is also then closed...]

[Hint: think complements. Note that $a < b$ is the same as $a < c < b$ for some (constant) c ...]

(*) (b) (The "other" proof) Use (a) and the Pasting Lemma to show that under the same hypotheses the functions

$$m(x) = \min\{f(x), g(x)\} \qquad \text{and} \qquad M(x) = \max\{f(x), g(x)\}$$

are (still!) both continuous.

34. Theorem 17.5 in Munkres shows that if $\mathcal{T} = \mathcal{T}(\mathcal{B})$ is a topology on X generated by the basis $\mathcal{B}$ and $A \subseteq X$, then $x \in \overline{A}$ precisely when every element of $\mathcal{B}$ that contains x meets A. Show, on the other hand, that no corresponding result holds for subbases $\mathcal{S}$. That is, we can have $A \subseteq X$ and an $x \in X$ so that every element of $\mathcal{S}$ that contains x meets A, but $x \notin \overline{A}$.

[Our favorite subbasis for the usual topology on $\mathbb{R}$ works, or you can get more inventive!]