Math 871 Problem Set 6

Starred (*) problems are due Thursday, October 9.

(*) 35. [Munkres, p.118, #8] For $i \in \mathbb{Z}_+$ let $a_i, b_i \in \mathbb{R}$ with $a_i > 0$ for all i, and let

$$f : \prod_i \mathbb{R} \to \prod_i \mathbb{R} \text{ be given by } f((x_i)_{i\in\mathbb{Z}_+}) = (a_i x_i + b_i)_{i\in\mathbb{Z}_+}$$

Show that f is a homeomorphism, when $\prod_i \mathbb{R}$ is given the product topology (on both the domain and codomain). What happens when $\prod_i \mathbb{R}$ has the box topology?

36. Find an example of subspaces $A, B \subseteq \mathbb{R}$ (giving $\mathbb{R}$ the usual topology) for which there is a continuous bijection

$$f : A \to B$$

whose inverse is **not** continuous.

(*) 37. Show that if X is a space with topology generated by a basis $\mathcal{B}$, then X is Hausdorff if and only if for every $x, y \in X$ with $x \neq y$, there are $B, B' \in \mathcal{B}$ with $x \in B$, $y \in B'$ and $B \cap B' = \emptyset$.

38. [Munkres, p.101, #11] Show that if $(X_\alpha, \mathcal{T}_\alpha)$ are Hausdorff for all α, then $\prod_\alpha X_\alpha$ is Hausdorff for both the product and box topologies.

39. Show that if $(X, \mathcal{T})$ and $(X, \mathcal{T}')$ are topologies on X, with $(X, \mathcal{T})$ Hausdorff, and $\mathcal{T} \subseteq \mathcal{T}'$, then $(X, \mathcal{T}')$ is Hausdorff.

(*) 40. Show that if $f, g : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ are both continuous, ănd $(Y, \mathcal{T}')$ is Hausdorff, then

$$C = \big\{x \in X : f(x) = g(x)\big\}$$

is a closed subset of X.

[Show that the complement is open....]

41. [restatement of Munkres, p.112, #13] Show that if $f, g : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ are both continuous, $A \subseteq X$ is a dense subset of X, $(Y, \mathcal{T}')$ is Hausdorff, and $f|_A = g|_A$, then $f = g$.

[To paraphrase, a continuous function to a Hausdroff space is uniquely determined by its values on a dense subset.]

[Hint: Problem #40 (!)]