**Math 871 Problem Set 7**

Starred (*) problems are due Thursday, October 16.

42. Show that if $h : (X\mathcal{T}) \to (Y, \mathcal{T}')$ is a homeomorphism, $A \subseteq X$, and $h(A) = B \subseteq Y$, then

$h|_A : (A, \mathcal{T}_A) \to (B, \mathcal{T}'_B)$ is also a homeomorphism.

43. [Munkres, p.144, Problem #2]

(*) (a) Show that if $p : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is continuous, and there is a continuous map $f : (Y, \mathcal{T}') \to (X, \mathcal{T})$ with $p \circ f = Id_Y$, then $p$ is a quotient map.

(*) (b) For $A \subseteq X$, a *retraction* of $X$ onto $A$ is a continuous map $r : X \to A$ with $r|_A = Id_A$. Show that a retraction is a quotient map.

44. [Munkres, p.152, Problem #1] Show that if $\mathcal{T} \subseteq \mathcal{T}'$ are topologies on $X$ and $(X, \mathcal{T}')$ is connected, then so is $(X, \mathcal{T})$.

45. Find an example of a topological space $(X, \mathcal{T})$ and $A \subseteq X$ so that $A$ is connected, but $\text{int}(A)$ isn't.

(Note: A result from class says that this čan't be done in $\mathbb{R}$. But it can be done in $\mathbb{R}^2$ ... or elsewhere!)

(*) 46. [Munkres, p.152, Problem #9] Suppose that $A \subsetneq X$ and $B \subsetneq Y$ are *proper* subsets of $X$ and $Y$. Show that if $(X\mathcal{T})$ and $(Y, \mathcal{T}')$ are both connected, then $(X \times Y) \setminus (A \times B)$ is a connected subset of $X \times Y$.

47. Give an example of a space $X$ and subset $A \subseteq X$ where $\text{int}(A)$ and $\text{cl}(A)$ are both connected, but $A$ is not.

(*) 48. [Munkres, p.152, Problem #11] Suppose that $p : X \to Y$ is a quotient map, $Y$ is connected, and for every $y \in Y$ the set $p^{-1}(\{y\})$ is a connected subset of $X$. Show that $X$ is connected.

[Hint: start with a potential separation for $X$; show that their images would separate $Y$.]