

Math 871 Problem Set 1

Starred (*) problems are due Thursday, Sept. 1.

1. [Munkres, p.14, #2 (part)] For each statement below, determine whether or not it is true. If true, show why; if not, give an example demonstrating this.
 - (*) (a) For any sets A, B , $A \setminus (A \setminus B) = B$.
 - (b) For any sets A, B, C , $\{A \subseteq B \text{ and } A \subseteq C\}$ if and only if $A \subseteq B \cap C$.
 - (c) For any sets A, B, C , $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$.
 - (d) For any sets A, B, C, D , $\{A \subseteq C \text{ and } B \subseteq D\}$ implies that $A \times C \subseteq B \times D$.
 - (e) For any sets A, B, C, D , $A \times C \subseteq B \times D$ implies that $\{A \subseteq C \text{ and } B \subseteq D\}$.
- (*) (f) For any sets A, B, C, D , $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
2. If $A, C \subseteq X$ and $B, D \subseteq Y$, show how to express $(A \times B) \setminus (C \times D)$ as a union of Cartesian products (i.e., sets of the form $U \times V$).
3. [Munkres, p.20, #1] Show that if $f : A \rightarrow B$ is a function, then
 - (a) If $A_0 \subseteq A$, then $A_0 \subseteq f^{-1}(f(A_0))$; the sets are equal, if f is injective.
 - (b) If $B_0 \subseteq B$, then $f(f^{-1}(B_0)) \subseteq B_0$; the sets are equal, if f is surjective.
4. [Munkres, p.20, #2 (part)] If $f : A \rightarrow B$ is a function, $A_0, A_1 \subseteq A$, and $B_0, B_1 \subseteq B$, then
 - (*) (a) $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1)$
 - (b) $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$
 - (*) (c) $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$, but equality does not always hold.
 - (d) $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$.
5. [Munkres, p.44, #7] If A and B are finite sets, show that the set $B^A = \{f : A \rightarrow B\}$ of all functions from A to B is also finite.
6. [Munkres, p.51, #5 (part)] For each of the following sets, determine whether or not it is countable:
 - (a) $A = \{f : \{0, 1\} \rightarrow \mathbb{Z}\}$, all functions from $\{0, 1\}$ to \mathbb{Z}
 - (b) $B = \{f : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+ : f(n) = 0 \text{ for all } n \geq 3\}$
 - (*) (c) $F = \{f : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+ : \text{there is } N \in \mathbb{Z}_+ \text{ with } f(n) = 0 \text{ for all } n \geq N\}$, all eventually-0 functions.
 - (d) $H = \{f : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+ : \text{there is } N \in \mathbb{Z}_+ \text{ with } f(n) = f(N) \text{ for all } n \geq N\}$, all eventually constant functions.
 - (e) $P = \{f : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+ : n > m \text{ implies } f(n) > f(m)\}$, all increasing functions.
 - (f) The set F of all finite subsets of \mathbb{N} .
7. Let $P(A) = \{B : B \subseteq A\}$ (the *power set* of A), and suppose $f : A \rightarrow P(A)$ is a function. Show that the set $B = \{a \in A : a \notin f(a)\}$ is not in the image of f ; conclude that f can never be surjective. What does this say about functions $g : P(A) \rightarrow A$?