Math 871 Exam 1

Your solutions are due Tuesday, October 20, uploaded to the 'Exam 1' file upload link found under the 'Exam 1' link on our Canvas Home page. In formulating your solutions you may consult your textbook (Munkres, *Topology*), your course notes, the instructor's classroom notes, and your (and the instructor's) problem set solutions. You may also seek clarification from the instructor, if any questions arise. You should not consult any other resource or communicate with any other person in the process of crafting your solutions.

57. Suppose that X and $A \subseteq X$ are (fixed) sets, and let
$$\mathscr{T} = \{U \subseteq X \ : \ U \cap A = \emptyset \ \underline{\text{or}} \ X \setminus U \text{ is countable}\}.$$
Show that $\mathscr{T}$ is a topology on X. What (familiar) name would we give to the subspace topology $\mathscr{T}_A$ on A?

58. If $f : X \to Y$ is a function and $\mathscr{B}$ is a basis for a topology on Y, show that
$$\mathscr{B}' = \{f^{-1}(B) \ : \ B \in \mathscr{B}\}$$
is a basis for a topology on X.

If f is surjective show, in addition, that $f : (X, \mathscr{T}(\mathscr{B}')) \to (Y, \mathscr{T}(\mathscr{B}))$ is a <u>quotient</u> <u>map</u>.

59. Give a topological space $(X, \mathscr{T})$ and a subset $A \subseteq X$, the <u>boundary</u> of A, ∂A, is defined as $\quad \partial A = \overline{A} \cap \overline{(X \setminus A)}$.

Show that if $f : (X, \mathscr{T}) \to (Y, \mathscr{T}')$ is continuous and $B \subseteq Y$, then $\partial(f^{-1}(B)) \subseteq f^{-1}(\partial B)$.

60. Show that there is no continuous bijection $f : [0,1) \to (0,1)$ (both with the usual subspace topologies).

[N.B.: there <u>are</u>, however, continuous injections <u>and</u> continuous surjections! (You do not need to show this...)]

Conclude that the spaces $X = (0,1) \subseteq \mathbb{R}$ and $Y = [0,1) \subseteq \mathbb{R}$ are <u>not</u> homeomorphic.