Math 871 Problem Set 10 (and last!)

Starred (**) problems are due Thursday, November 12.

78. Show that if $f, g : X \to Y$ and $h, k : Z \to W$ satisfy $f \simeq g$ and $h \simeq k$, then $f \times h, g \times k : X \times Z \to Y \times W$ are homotopic.

79. [Hatcher, p.18, Problem #3(c)] Show that if maps $f, g : X \to Y$ are homotopic and f is a homotopy equivalence, then g is also a homotopy equivalence.

(**) 80. [Munkres, p.330, Problem #3(c)] Show that if X is a contractible space, then any two maps $f, g : Y \to X$ are homotopic.

81. [Munkres, p.330, Problem #3(d)] Show that if X is a contractible space, <u>and</u> Y is path connected, then any two maps $f, g : X \to Y$ are homotopic.

(**) 82. Show that the cone on a circle $cS^1 = S^1 \times I / \sim$, where $(x, 1) \sim (y, 1)$ for $x, y \in S^1$, with the quotient topology, is homeomorphic to the unit disk $D^2 = \{\vec{x} \in \mathbb{R}^2 : ||\vec{x}|| \leq 1\}$. [Hint: build a map from compact to Hausdorff...]

83. Show that if X is Hausdorff and $A \subseteq X$, and $r : X \to X$ is a retraction of X onto A (see Problem #51 of Problem Set 7), then A is a closed subset of X. [Hint: $x \notin A \Leftrightarrow r(x) \neq x$ ...]

(**) 84. [Hatcher, p.19, Problem #9] Show that if X is contractible, and $A \subseteq X$ is a retract of X (i.e., there is a retraction from X onto A...), then A is contractible.

85. [Munkres, p.186, Problem #11a] Show that if $p : X \to Y$ is a quotient map, and Z is a locally compact Hausdorff space, then $f = p \times \mathrm{Id}_Z : X \times Z \to Y \times Z$, given by $f(x, z) = (p(x), z)$, is also a quotient map.

Note: a space is *locally compact* if for every point $x \in X$ there is a compact subset $C \subseteq X$ with $x \in C$ and a neighborhood $x \in U \in \mathcal{T}$ with $x \in U \subseteq C$. E.g., if X is compact then $C = X$ works!

[This problem is mostly so that I can a some point in the future <u>quote</u> this result at you (when $Z = [0, 1]$)... See the very detailed hints in Munkres' text!]