Math 871 Problem Set 5

Starred (**) problems are due Thursday, September 24.

(**) 32. Giving $\mathbb{R}$ the usual (metric) topology, show that if $f : (X, \mathcal{T}) \to \mathbb{R}$ and $g : (X, \mathcal{T}) \to \mathbb{R}$ are both continuous, then the functions $m, M : (X, \mathcal{T}) \to \mathbb{R}$, given by

$$m(x) = \min\{f(x), g(x)\} \qquad M(x) = \max\{f(x), g(x)\}$$

are also both continuous.

[Hint: using the subbasis $\mathcal{S} = \{(a, \infty) \ : \ a \in \mathbb{R}\} \cup \{(-\infty, a) \ : \ a \in \mathbb{R}\}$ cuts down on your work...]

33. (a) Show that $\{(x, y) \in \mathbb{R}^2 \ : \ x + y > a\} = \bigcup_c (c, \infty) \times (a - c, \infty)$ and $\{(x, y) \in \mathbb{R}^2 \ : \ x + y < a\} = \bigcup_c (-\infty, c) \times (-\infty, a - c)$ (so both are open sets).

(b) Show that the function $A : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ (with the usual (product) toplogies) given by $A(x, y) = x + y$ is continuous. [Part (a) helps... see previous hint!]

(c) Use a previous problem (!) to conclude that if $f, g : (X, \mathcal{T}) \to (\mathbb{R}, \text{usual})$ are both continuous , then the function $h : (X, \mathcal{T}) \to (\mathbb{R}, \text{usual})$ given by $h(x) = f(x) + g(x)$ is also continuous.

34. For $A, B \subseteq X$ with $(X, \mathcal{T})$ a topological space, if A is open in X and B is closed in X, then $A \setminus B$ is open and $B \setminus A$ is closed.

35. 35. [Munkres, p.101, #1] Show that if $A, B, A_\alpha \subseteq X$, where $(X, \mathcal{T})$ is a topological space, then

(a) If $A \subseteq B$ then $\overline{A} \subseteq \overline{B}$ (b) $\overline{A} \cup \overline{B} = \overline{A \cup B}$

(c) $\cup_\alpha \overline{A_\alpha} \subseteq \overline{\cup_\alpha A_\alpha}$; in general, equality does not hold.

(**) 36. Show that if $A \subseteq X$ and X has two topologies $\mathcal{T} \subseteq \mathcal{T}'$, then if $x \in X$ is a limit point of A w.r.t. $\mathcal{T}'$, then it is also a limit point of A w.r.t. $\mathcal{T}$.

37. Find the closure of the set $A = \{1 - \frac{1}{n} : n \in \mathbb{Z}_+\} \subseteq \mathbb{R}$, when $\mathbb{R}$ has the

(a) finite complement topology

(b) infinite (open) ray to the right topology

(c) discrete topology

(d) *lower limit topology*, generated by the basis $\mathcal{B} = \{[a, b) : a, b \in \mathbb{R}\}$

(e) countable complement topology.

(**) 38. [Limit points don't like cts fcns] Show, by example(s), that if $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is continuous and $A \subseteq X$, then $x \in A'$ <u>need</u> <u>not</u> imply that $f(x) \in (f(A))'$, and $f(x) = y \in (f(A))'$ <u>need</u> <u>not</u> imply that there is a $z \in A'$ with $f(z) = y$. [I don't know if a single example can establish both...]

39. Theorem 17.5 in Munkres shows that if $\mathcal{T} = \mathcal{T}(\mathcal{B})$ is a topology on X generated by the basis $\mathcal{B}$ and $A \subseteq X$, then $x \in \overline{A}$ precisely when every element of $\mathcal{B}$ that contains x meets A. Show, on the other hand, that no corresponding result holds for <u>sub</u>bases $\mathcal{S}$. That is, we can have $A \subseteq X$ and an $x \in X$ so that every element of $\mathcal{S}$ that contains x meets A, <u>but</u> $x \notin \overline{A}$.