Math 871 Problem Set 9

Starred (**) problems are due Thursday, November 5.

69. Show that if X is limit point compact, and A is a closed subset of X, then A is limit point compact.

70. [Munkres, p.194, #12] Show that if $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is continuous and *open* and X is first countable, then $A = f(X) \subseteq Y$, with the subspace topology, is first countable.

(**) 71. [Munkres, p.194, #5(a),#6] Show that if $(X, \mathcal{T})$ is a *metrizable*, separable space, then X is second countable. Conclude that $\mathbb{R}$ with the lower limit topology is <u>not</u> metrizable.

72. For $(X, \mathcal{T})$ a topological space and $A \subseteq X$, $x \in X$ is called a *condensation point* of A if for every $U \in \mathcal{T}$ with $x \in U$, $U \cap A$ is <u>uncountable</u>.

    Show that if X is second countable, then every uncountable subset of X has a condensation point.

73. [Munkres, p.205, #1] Show that a closed subset $A \subseteq X$ of a normal (= T_1 plus T_4) space $(X, \mathcal{T})$ is normal.

74. Show by example that if $\mathcal{T}' \subseteq \mathcal{T}$ are topologies on X and $(X, \mathcal{T})$ is regular (= T_3 plus T_1), we cannot conclude that $(X, \mathcal{T}')$ is regular, even if it is T_1. Conclude that the continuous image of a regular space need not be regular. [Cheap route: think discrete topology...]

75. [Munkres, p.199, # 6] If $p : X \to Y$ is a continuous, closed, surjective map and X is normal (i.e, T_4 and T_1), then Y is normal. [Hint: see Munkres' hint!]

(**) 76. [Munkres, p.199, # 1] Show that if $(X, \mathcal{T})$ is regular, then for every $x, y \in X$ with $x \neq y$, there are $U, V \in \mathcal{T}$ so that $x \in U, y \in V$ and $\overline{U} \cap \overline{V} = \emptyset$.

(**) 77. [Munkres, p.205, #2] Show that if $(X_\alpha, \mathcal{T}_\alpha)$ are <u>non</u>-<u>empty</u> (!) spaces and $\prod_\alpha X_\alpha$ is normal (in the product topology), then each X_α is normal. [Remember, though, the converse is false! And note that problem #75 doesn't help...]