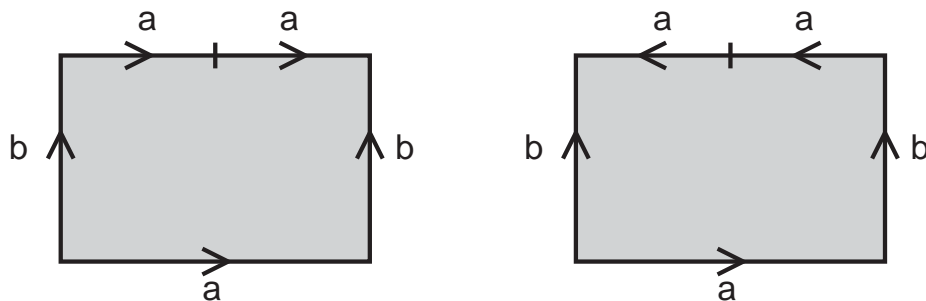


Math 872 Algebraic Topology

Problem Set # 5

Starred (*) problems due Thursday, March 8

- 22.** Find a Δ -complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $S^1 \times \{1\}$ to $S^1 \times \{0\}$ by a map representing 2 times the generator of $\pi_1(S^1)$. (See figure below.)



- (*) **23.** Find a Δ -complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $s^1 \times \{1\}$ to $s^1 \times \{0\}$ by a map representing -2 times the generator of $\pi_1(S^1)$. (See figure above.)
- (*) **24.** Compute the simplicial homology groups of the 3-simplex Δ^3 (that is, the Δ -complex obtained from 4 vertices by gluing on 6 1-simplices, 4 2-simplices and a single 3-simplex in the “obvious” way).
- 25.** Regarding the n -simplex $X = \Delta^n$ as a Δ -complex in the natural way, show that if $A \subseteq X$ is a subcomplex with $H_{n-1}(A) \neq 0$, then $A = \partial\Delta^n$. (Hint: show that an $(n-1)$ -cycle for A (and hence for X) must either be 0 or have non-zero coefficient for every $(n-1)$ -dimensional face of X .)
- 26.** Show that if $A \subseteq X$ is a retract of X , then the inclusion map $\iota : A \rightarrow X$ induces an injection on all singular homology groups.