Straight-line homotopies:

Any two maps $f, g : X \to \mathbb{R}^n$ are homotopic:

$H(x,t) = (1-t)f(x) + tg(x)$

is continuous, since "it is constructed out of continuous functions" in a manner which we know preserves continuity.

Note that if $f(x_0) = g(x_0) = y_0$, then $H(x_0,t) = (1-t)y_0 + ty_0 = y_0$. So the homotopy is relative to the set $A = \{x \in X : f(x) = g(x)\}$.

So, for example, any two paths $\alpha, \beta : I \to \mathbb{R}^n$ in $\mathbb{R}^n$ (or any convex subset of $\mathbb{R}^n$) between the same endpoints are homotopic rel endpoints.

The same (by composing with a homeomorphism) is true of any space homeomorphic to (a convex subset of) $\mathbb{R}^n$.

Grafting a "tail" onto a path also does not change it's homotopy class:

Since $\gamma * \overline{\gamma}$ is homotopic, rel endpoints, to a constant map c_0,

$$\alpha * \beta \simeq \alpha * c_0 * \beta \simeq \alpha * \gamma * \overline{\gamma} * \beta \simeq (\alpha * \gamma) * (\overline{\gamma} * \beta)\ .$$