A non-trivial presentation of the trivial group

$G = \langle a, b \mid bbab^{-1}a^{-1}, aaba^{-1}b^{-1} \rangle$
$= \langle a, b \mid aba^{-1} = b^2, bab^{-1} = a^2 \rangle$

Then:

$b = (aaba^{-1}b^{-1})b = aaba^{-1}(b^{-1}b) =$
$aaba^{-1} = aabbb^{-1}a^{-1} =$
$aabb(aba^{-1})^{-1}(bbab^{-1}a^{-1})^{-1}(aba^{-1})b^{-1}a^{-1} =$
$aabbab^{-1}a^{-1}aba^{-1}b^{-1}b^{-1}aba^{-1}b^{-1}a^{-1} =$
$aabbab^{-1}a^{-1}aba^{-1}b^{-1}b^{-1}aba^{-1}b^{-1}a^{-1} =$
$aabbab^{-1}ba^{-1}b^{-1}b^{-1}aba^{-1}b^{-1}a^{-1} =$
$aabbaa^{-1}b^{-1}b^{-1}aba^{-1}b^{-1}a^{-1} =$
$aabbb^{-1}b^{-1}aba^{-1}b^{-1}a^{-1} = aabb^{-1}aba^{-1}b^{-1}a^{-1} =$
$aaaba^{-1}b^{-1}a^{-1} = aa^{-1} = 1$

Similarly, $a = 1$ in G. So every word in a and b is 1, so $G = \{1\}$.

$\langle a, b, c \mid aba^{-1} = b^2, bcb^{-1} = c^2, cac^{-1} = a^2 \rangle$ is also a presentation for the trivial group, but the proof is much more involved.

On the other hand, $H = \langle a, b \mid aba = b^2, bab = a^2 \rangle$ isn't trivial, since
$\varphi : F(a, b) \to \mathbb{Z}_3$ defined by $\varphi(a) = 1, \varphi(b) = 2$ is onto and satisfies $\varphi(aba) = 4 = 1 = 4 = \varphi(b^2)$ and $\varphi(bab) = 5 = 2 = \varphi(a^2)$, so descends to a surjective homomorphism $\vartheta : H \to \mathbb{Z}_3$. So $H \neq \{1\}$.