Another application of homotopy extension:

If S is a contractible CW-cplx and $x_0 \in X^{(0)}$, then X strong def. retracts to $\{x_0\}$.

By hypoth., the inclusion map $\iota : \{x_0\} \hookrightarrow X$ is a h.e., With $r : X \to \{x_0\}$ the retraction, and $H : X \times I \to X$ a homotopy from $H_0 = I_X$ to $H_1 = \iota \circ r$. This almost a strong def. retraction except that we don't have $H_t|_A$ is the identity; that is, we don't have $H_t(x_0) = x_0$ for all t. Instead, the map $\gamma : t \mapsto H(x_0, t)$ describes a loop in X based at x_0.

We need to deform the homotopy H to a homotopy L so that at the ends $t = 0, 1$ it still does what we want (interpolating betwen the identity and constant maps), but $L(x_0, t) = x_0$ for all t. Essentially, then, we want to deform γ to the constant map. But this we can do, because contractible spaces are simply-connected; $\gamma \simeq *$. What we want to do, then, is to extend this homotopy

$J : \{x_0\} \times I \times I \to X$, or rather, the map $J' : A \times I = (\{x_0\} \times I \cup X \times \{0, 1\}) \times I \to X$ given by J on the first half and $J'(x, 0, t) = x, J'(x, 1, t) = x_0$ on the second, to $X \times I \times I$,. But: A is a subcomplex of the CW-complex $X \times I$, and we have a map $H : X \times I \times \{0\} \to X$, and a homotopy $J' : A \times I \to X$ which agrees with H on $A \times \{0\}$. So by H.E.P. there is a homotopy $K : (X \times I) \times I \to X$ extending H and J'. But then $L : X \times I \to X$ defined by $L(x, t) = K(x, t, 1)$ satisfies $L(x, 0) = K(x, 0, 1) = J'(x, 0, 1) = x$, $L(x, 1) = J'(x, 1, 1) = x_0$, and $L(x_0, t) = K(x_0, t, 1) = J(t, 1) = x_0$, for all x and t. That is, we have a homotopy from the identity to the constant map, which leaves x_0 fixed. So we have built our strong deformation retraction.