Covering spaces:

The projective plane $\mathbb{R}P^2$ has $\pi_1 = \mathbb{Z}_2$. It is also the quotient of the simply-connected space S^2 by the antipodal map, which, together with the identity map, forms a group of homeomorphisms of S^2 which is isomorphic to $\mathbb{Z}_2$. The fact that $\mathbb{Z}_2$ has this dual role to play in describing $\mathbb{R}P^2$ is no accident; codifying this relationship requires the notion of a covering space.

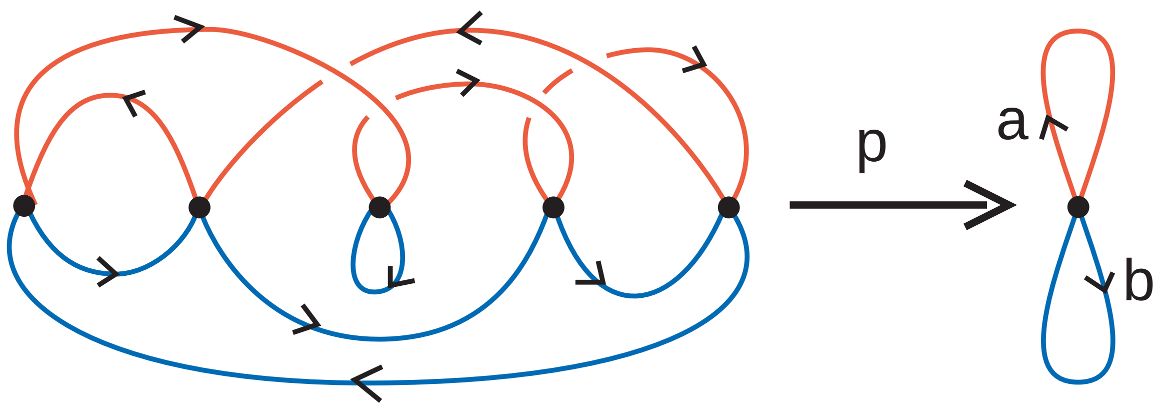
The quotient map $q : S^2 \to \mathbb{R}P^2$ is an example of a *covering map*. A map $p : E \to B$ is called a covering map if for every point $x \in B$, there is a neighborhood $\mathcal{U}$ of x (an *evenly covered neighborhood*) so that $p^{-1}(\mathcal{U})$ is a disjoint union $\mathcal{U}_\alpha$ of open sets in E, each mapped homeomorphically onto $\mathcal{U}$ by (the restriction of) p . B is called the *base space* of the covering; E is called the *total space*.

The quotient map q is an example; (the image of) the complement of a great circle in S^2 will be an evenly covered neighborhood of any point it contains.

The disjoint union of 42 copies of a space, each mapping homeomorphically to a single copy, is an example of a *trivial covering*.

The famous exponential map $p : \mathbb{R} \to S^1$ given by $t \mapsto e^{2\pi i t} = (\cos(2\pi t), \sin(2\pi t))$. The image $J \subseteq S^1$ of any interval (a, b) of length less than 1 will have inverse image the disjoint union of the intervals $(a + n, b + n)$ for $n \in \mathbb{Z}$.

We can build many finite-sheeted (every point inverse is finite) coverings of a bouquet of two circles, by assembling n points over the vertex, and then, on either side (the red/blue sides?), connecting the points by n (oriented) arcs, one with one red/blue arcs going in/out of each vertex. By choosing orientations on each 1-cell of the bouquet, we can build a covering map by sending the vertices above to the vertex, and the arcs to the one cells, homeomorphically, respecting the orientations. We can build infinite-sheeted coverings in much the same way.



Covering spaces of more "interesting" graphs can be assembled similarly.