Lifting properties:

Covering spaces of a (suitably nice) space X have a very close relationship to $\pi_1(X, x_0)$.

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Homotopy Lifting Property: If $p : \widetilde{X} \to X$ is a covering map, $H : Y \times I \to X$ is a homotopy $H(y, 0) = f(y)$ a a homotopy, $H(y, 0) = f(y)$, and $f: Y \to X$ is a *lift* of f (i.e., $p \circ f = f$), then there space X h
 $p : \widetilde{X} \to$
 $\vdots Y \to \widetilde{X}$
 $\vdots \to \widetilde{f}(\omega)$ **Lifting properties:**
Covering spaces of a (suitably n
Homotopy Lifting Propert;
a homotopy, $H(y, 0) = f(y)$, an
is a unique lift \widetilde{H} of H with \widetilde{H} $(y, 0) = f(y)$.

In particular, applying this property in the case $Y = \{*\}$, where a homotopy $H \cdot \{*\} \times I \to X$ is really just a a path $\gamma \cdot I \to X$ we have the $H: \{*\} \times I \to X$ is really just a a path $\gamma: I \to X$, we have the
Poth Lifting Property: Given a covering map $n: \tilde{Y} \to Y$ a homotopy, $H(y, 0) = f(y)$, and $f : Y \to X$ is a *lift* of f
is a unique lift \widetilde{H} of H with $\widetilde{H}(y, 0) = \widetilde{f}(y)$.
In particular, applying this property in the case $Y = \{*\}$
 $H : \{*\} \times I \to X$ is really just a a path **Path Lifting Property:** Given a covering map $p : X \to X$, a path $\gamma : I \to X$ is a unique lift H of H with H
In particular, applying this pro
 $H: \{*\} \times I \to X$ is really just a
Path Lifting Property: Giv
with $\gamma(0) = x_0$, and a point \tilde{x}
 $\tilde{\gamma}(0) - \tilde{x}_0$ $(y, 0) = f(y)$.

perty in the case $Y = \{*\}$, where a hor

a a path $\gamma : I \to X$, we have the

en a covering map $p : \tilde{X} \to X$, a pa
 $p \in p^{-1}(x_0)$, there is a unique path $\tilde{\gamma}$ with $\gamma(0) = x_0$, and a point $\tilde{x}_0 \in p^{-1}(x_0)$, there is a unique path $\tilde{\gamma}$ lifting γ with $\begin{array}{c} \mathbf{I}_1 \ L \ \mathbf{F} \ \text{w} \ \widetilde{\gamma} \ \Lambda \end{array}$ (1) partic
 $I: \{*\} >$
 ath Li
 $\gamma(0) = \tilde{x}$
 $\gamma(0) = \tilde{x}$ $\widetilde{\gamma}(0) = \widetilde{x}_0.$ **Path Li**
with $\gamma(0)$
 $\widetilde{\gamma}(0) = \widetilde{x}_0$
An imme
If $p : (\widetilde{X}, \widetilde{X})$

An immediate consequence:

 $(\tilde{x}_0) \rightarrow (X, x_0)$ is a covering map, then the induced homomorphism with $\gamma(0)$
 $\widetilde{\gamma}(0) = \widetilde{x}_0.$

An immed

If $p : (\widetilde{X}, \widetilde{a})$
 $p_* : \pi_1(\widetilde{X}, \widetilde{a})$ $\frac{1}{x}$ is $\frac{1}{x}$ π_0) $\to \pi_1(X, x_0)$ is injective. $\gamma(0) = x_0.$

An immediate consequen

If $p : (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)$ is
 $p_* : \pi_1(\widetilde{X}, \widetilde{x}_0) \to \pi_1(X, x_0)$
 Proof: $\gamma : (I, \partial I) \to (\widetilde{X}, H) \to (I \times I, \partial I \times I) \to (X, H)$ $\begin{pmatrix} 3 \ 2 \end{pmatrix}$

0) a loop with $p_*(\gamma) = 1$ in $\pi_1(X, x_0)$. There is
constant interpolating between $n \circ \gamma$ and the constant $H: (I \times I, \partial I \times I) \to (X, x_0)$ interpolating between $p \circ \gamma$ and the constant path. By If $p: (X, \tilde{x}_0) \to (X, x_0)$ is a covering ma
 $p_* : \pi_1(\tilde{X}, \tilde{x}_0) \to \pi_1(X, x_0)$ is injective.
 Proof: $\gamma: (I, \partial I) \to (\tilde{X}, \tilde{x}_0)$ a loop with
 $H: (I \times I, \partial I \times I) \to (X, x_0)$ interpolati

homotopy lifting, there is a homotop homotopy lifting, there is a homotopy H from γ to the lift of the constant map at x_0 . $p_* : \pi_1(\tilde{X}, \tilde{x}_0) \to \pi_1(X, x_0)$ is injective.
 Proof: $\gamma : (I, \partial I) \to (\tilde{X}, \tilde{x}_0)$ a loop with $p_*([\gamma]) = 1$ in $\pi_1(X, x_0)$. There is
 $H : (I \times I, \partial I \times I) \to (X, x_0)$ interpolating between $p \circ \gamma$ and the constant path. By
 Proof: $\gamma : (I, \partial I) \to (\tilde{X}, \tilde{x}_0)$ a loop with $p_*([\gamma]) = 1$ in $\pi_1(X, x_0)$. Then $H : (I \times I, \partial I \times I) \to (X, x_0)$ interpolating between $p \circ \gamma$ and the constantion
homotopy lifting, there is a homotopy \tilde{H} from γ to th from $H(0,0), H(1,0) = \gamma(0) = \gamma(1) = \tilde{x}_0$, so are the constant map at \tilde{x}_0 . So the lift $H: (I \times I, \partial I \times I) \to (X, x_0)$ a loop with $p_{*}([f])$
 $H: (I \times I, \partial I \times I) \to (X, x_0)$ interpolating betw

homotopy lifting, there is a homotopy \widetilde{H} from γ

The vertical sides $s \mapsto \widetilde{H}(0, s), \widetilde{H}(1, s)$ are also

from $\$ at the bottom is the constant map at \tilde{x}_0 . So H represents a null-homotopy of γ , so $\begin{aligned} \n\text{In} \cdot (\mathbf{Y} \times \mathbf{Y}, \mathbf{y} \times \mathbf{Y}) \n\text{homotopy lifting} \\ \n\text{The vertical side} \\ \n\text{from } \widetilde{H}(0,0), \widetilde{H}(0,0) \n\end{aligned}$ at the bottom is $[\gamma] = 1$ in $\pi_1(\widetilde{X}, \widetilde{X})$ $\frac{1}{x}$, $\frac{1}{x}$ $\bigg(0\bigg)$.

Even more, $p_*(\pi_1(\widetilde{X},$ whose lifts to paths \widetilde{x}
 \pm (0))) $\subseteq \pi_1(X, x_0)$ is precisely the elements given by loops at x_0 ,
arting at \widetilde{x}_0 are loops. If α lifts to a loop $\widetilde{\alpha}$ then $n \circ \widetilde{\alpha} = \alpha$ Even more, $p_*(\pi_1(\tilde{X}, \tilde{x}_0)) \subseteq \pi_1(X, x_0)$ is precisely the elements given by loops as whose lifts to paths starting at \tilde{x}_0 , are loops. If γ lifts to a loop $\tilde{\gamma}$, then $p \circ \tilde{\gamma}$ so $p_0(\tilde{\chi}) = [\gamma]$ of p_0 whose lifts to paths starting at \tilde{x}_0 , are loops. If γ lifts to a loop $\tilde{\gamma}$, then $p \circ \tilde{\gamma} = \gamma$, Even more, $p_*(\pi_1(\tilde{X}, \tilde{x}_0))) \subseteq \pi_1(X, x_0)$ is precisely
whose lifts to paths starting at \tilde{x}_0 , are loops. If ∞
so $p_*([\tilde{\gamma}]) = [\gamma]$. If $p_*([\tilde{\gamma}]) = [\gamma]$, then $\gamma \simeq p \circ \tilde{\gamma}$
to a homotony b/w the lift of γ a so $p_*(\tilde{\gamma}) = [\gamma]$. If $p_*(\tilde{\gamma}) = [\gamma]$, then $\gamma \simeq p \circ \tilde{\gamma}$ rel endpoints; the homotopy lifts Even more, $p_*(\pi_1(\tilde{X}, \tilde{x}_0)) \subseteq \pi_1(X, x_0)$ is precisely the elements given by loops
whose lifts to paths starting at \tilde{x}_0 , are loops. If γ lifts to a loop $\tilde{\gamma}$, then $p \circ \tilde{\gamma}$
so $p_*([\tilde{\gamma}]) = [\gamma]$. If $p_*([\tilde$ to a homotopy b/w the lift of γ at \tilde{x}_0 and the lift of $p \circ \tilde{\gamma}$ at \tilde{x}_0 (which is $\tilde{\gamma}$, since $\begin{array}{c} \text{F} \text{w} \text{se} \ \text{t} \text{e} \ \widetilde{\gamma} \end{array}$ Figure 11
 $\frac{1}{\sqrt{2}}$
 $\tilde{\gamma}(0) = \tilde{x}_0$ and lifts are unique). So the lift of γ is a loop, as desired.

Proof of H.L.P.: lift maps a little bit at a time! Cover X by evenly covered open sets \mathcal{U}_i . For each fixed $y \in Y$, since I is compact and the sets $H^{-1}(\mathcal{U}_i)$ form an open cover of $Y \times I$, the Tube Lemma provides an open neighborhood \mathcal{V}_y of y in Y and finitely many $p^{-1}U_i$ which cover $\mathcal{V}_y \times I$. **Proof** of H.L.P.: lift maps a little bit at a time! Cover X by evenly covered open
sets \mathcal{U}_i . For each fixed $y \in Y$, since I is compact and the sets $H^{-1}(\mathcal{U}_i)$ form an open
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cover of $Y \times I$, the Tube Lemma provides an open nei
finitely many $p^{-1}\mathcal{U}_i$ which cover $\mathcal{V}_y \times I$.
To define $\widetilde{H}(y, t) = \widetilde{H}_y(t)$, cut $\{y\} \times I$

H. Starting from the left, we have (inductively) a lift $H_y(t_j)$ of the left endpoint t_j of cover of
finitely
To defin
 $H.$ Star
 I_j to \widetilde{X}
 \widetilde{H} (4) I_j to X, and a homeo $h_j: U_j \to \mathbb{R}$ component of its inverse image of U_j containing $\begin{array}{c} {\rm{h \hskip 0.1em} n} \ {\rm{H}} \ I_j \ {\tilde H} \ {\rm{h \hskip 0.1em} n} \end{array}$ intely many $p^{-1}U_i$ which cover $V_y \times I$.

b) define $\widetilde{H}(y,t) = \widetilde{H}_y(t)$, cut $\{y\} \times I$ into pieces I_j , each mapp

. Starting from the left, we have (inductively) a lift $\widetilde{H}_y(t_j)$ of to \widetilde{X} , and a homeo $H_y(t_j)$. Then define H_y on I_j to be $h_j \circ H_y$. By induction, $H(y,t)$ is defined for all t (and y). This definition is independent of the partition $\{y\} \times I$, by the usual process of taking the union of the partitions, and noticing that the choice of h_j is unique. (If we change the open cover, we can compare using the intersections; the choice of h_j 's $\widetilde{H}_y(t_j)$. Then define \widetilde{H}_y on I_j to be $h_j \circ H_y$. By induction, $\widetilde{H}(y,t)$ is defined for all t (and y). This definition is independent of the partition $\{y\} \times I$, by the usual process of taking the union o $\begin{align} H & \text{(a} \text{ of } \text{w} \text{)} \ \text{w} & \text{w} \ \widetilde{H} & \text{cc} \end{align}$ \tilde{H} is continuous since for y near y_0 we can use the same partitions and the same open cover (because of our tube lemma condition), which means that we use the same maps h_i to lift; the pasting lemma implies continuity.

So, for example, if we build a 5-sheeted cover of the bouquet of 2 circles, as before, (after choosing a maximal tree upstairs) we can read off the images of the generators of the fundamental group of the total space; we have labelled each edge by the generator it traces out downstairs, and for each edge outside of the maximal tree chosen, we read from basepoint out the tree to one end, across the edge, and then back to the basepoint in the tree. In our example, this gives:

 $\langle \alpha, a, a \alpha b^{-1}, b \alpha b a^{-1}, b \alpha a, b \alpha^{-1} b a b^{-1}, b \alpha b^{-1} b^{-1} \rangle$

This is (from its construction) a copy of the free group on 6 letters, in the free group $F(a, b)$. In a similar way, by explicitly building a covering space, we find that the fundamental group of a closed surface of genus 3 is a subgroup of the fundamental group of the closed surface of genus 2.

The cardinality of a point inverse $p^{-1}(y)$ is, by the evenly covered property, constant on (small) open sets, so the set of points of x whose point inverses have any given cardinality is open. Consequently, if X is connected, this number is constant over all The cardinality of a point inverse $p^{-1}(y)$ is, by the evenly covered pron (small) open sets, so the set of points of x whose point inverses cardinality is open. Consequently, if X is connected, this number is of X, and i

The number of sheets of a covering map can also be determined from the fundamental groups: of X, and is called the nu

The number of sheets of a

groups:
 Proposition: If X and \widetilde{X}

map equals the index of t

 are path-connected, then the number of sheets of a covering The number of sheets of a covering map can also be degroups:
 Proposition: If X and \widetilde{X} are path-connected, then the map equals the index of the subgroup $H = p_*(\pi_1(\widetilde{X},$ $\det \widetilde{x}$ (b) in $G = \pi_1(X, x_0)$.

—bf Proof: Choose loops $\{g_{\alpha} = [\gamma_{\alpha}]\}\$, one in each of the (right) cosets of H in G. Lift **Proposition:** If X and \widetilde{X} are path-connected, then the number of sheets of map equals the index of the subgroup $H = p_*(\pi_1(\widetilde{X}, \widetilde{x}_0)$ in $G = \pi_1(X, x_0)$ — bf Proof: Choose loops $\{g_\alpha = [\gamma_\alpha]\}$, one in each of them to loops based at \tilde{x}_0 ; they will have distinct endpoints. (If $\tilde{\gamma}_1(1) = \tilde{\gamma}_2(1)$, then **Proposition:** If X and X are path-connected, then the number of sheets of a covering
map equals the index of the subgroup $H = p_*(\pi_1(\tilde{X}, \tilde{x}_0)$ in $G = \pi_1(X, x_0)$.
—bf Proof: Choose loops $\{g_{\alpha} = [\gamma_{\alpha}]\}$, one in eac map equals the index of

—bf Proof: Choose loop

them to loops based at

by uniqueness of lifts, γ

an element of $p_*(\pi_1(\widetilde{X},\$ in $n^{-1}(x_0))$ is the endno $\begin{array}{c} \n\mathbf{r} \\
\mathbf{s} \\
\mathbf{r} \\
\mathbf{r} \\
\mathbf{r}\n\end{array}$ (0) , so they are in the same coset.) Conversely, every point in p−1(x₀) is the endpoint of one of the endpoints. (If $\tilde{\gamma}_1(1) = \tilde{\gamma}_2(1)$, by uniqueness of lifts, $\gamma_1 * \overline{\gamma_2}$ lifts to $\tilde{\gamma}_1 * \overline{\gamma_2}$, so it lifts to a loop, so $\gamma_1 * \overline{\gamma_2}$ represent a path $p_*(\pi_1(\til$ in $p^{-1}(x_0)$ is the endpoint of one of these lifts, since we can construct a path $\tilde{\gamma}$ from $\frac{1}{2}$ the big as in \widetilde{x} ports on 1 tool. Choose hoops $\{y_{\alpha} - \gamma_{\alpha}\}\}$, one in a
nem to loops based at \tilde{x}_0 ; they will have dis
y uniqueness of lifts, $\gamma_1 * \overline{\gamma_2}$ lifts to $\tilde{\gamma}_1 * \tilde{\gamma}_2$, s
n element of $p_*(\pi_1(\tilde{X}, \tilde{x}_0))$, so they ar \tilde{x}_0 to any such point y, giving a loop $\gamma = p \circ \tilde{\gamma}$ representing an element $g \in \pi_1(X, x_0)$. But then $g = hg_\alpha$ for some $h \in H$ and α , so γ is homotopic rel endpoints to $\eta * \gamma_\alpha$ for by uniqueness of mts, γ₁ * γ₂ mts to γ₁ * γ₂, so it mts to a loop, so γ₁ * γ₂ represents
an element of $p_*(\pi_1(\tilde{X}, \tilde{x}_0))$, so they are in the same coset.) Conversely, every point
in $p^{-1}(x_0)$ is the end in $p^{-1}(x_0)$ is the endpoint of one of these lifts, since we can construce \tilde{x}_0 to any such point y, giving a loop $\gamma = p \circ \tilde{\gamma}$ representing an element But then $g = hg_\alpha$ for some $h \in H$ and α , so γ is homotopi is homotopic rel endpoints to $\tilde{\eta}$, which is a loop, followed by the lift $\tilde{\gamma}_{\alpha}$ of γ_{α} starting \tilde{x}_0 to any such poi

But then $g = hg_\alpha$

some loop η repres

is homotopic rel es

at \tilde{x}_0 . So $\tilde{\gamma}$ and $\tilde{\gamma}$ at \widetilde{x}_0 . So $\widetilde{\gamma}$ and $\widetilde{\gamma}_{\alpha}$ have the same value at 1. some loop η representing h. But then lifting these based at \tilde{x}_0 , by homotopy lifting, $\tilde{\gamma}$ is homotopic rel endpoints to $\tilde{\eta}$, which is a loop, followed by the lift $\tilde{\gamma}_{\alpha}$ of γ_{α} starting at \tilde

Therefore, lifts of representatives of coset representatives of H in G give a 1-tocardinality.

The path lifting property (because $\pi([0, 1], 0) = \{1\}$) is a special case of the **lifting**
containing If $x : (\tilde{Y}, \tilde{x}) \to (X, x)$ is a severing map and $f : (X, y) \to (X, x)$ is a The path lifting propriation: If $p : (\tilde{X}, \text{map})$ where Y is path $\frac{\partial e}{\partial x}$ f_0 \rightarrow (X, x_0) is a covering map, and $f : (Y, y_0) \rightarrow (X, x_0)$ is a connected and locally path-connected then there is a unique map, where Y is path-connected and locally path-connected, then there is a unique
lift $\tilde{f} \cdot (Y, y_0) \rightarrow (\tilde{X}, \tilde{x}_0)$ of f (i.e. $f = n \circ \tilde{f}$) $\Leftrightarrow f (\pi_1(Y, y_0)) \subset n (\pi_1(\tilde{X}, \tilde{x}_0))$ \int in the lifting property (because $\pi([0, 1], 0) = \{1\}$) is a special case of
 rion: If $p : (\tilde{X}, \tilde{x}_0) \to (X, x_0)$ is a covering map, and $f : (Y, y_0) \to (0, \tilde{X}, \tilde{x}_0)$ where *Y* is path-connected and locally path-connect $\begin{align} \frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} \end{align}$ o) of f (i.e., $f = p \circ f$) $\begin{array}{c} \text{th} \ \text{N} \ \text{is} \ \widetilde{x} \end{array}$ $\pi_1(Y, y_0)) \subseteq p_*(\pi_1(X, \widetilde{x}_0))$. If f exists, then $f_* = p_* \circ f$ $f_*,$ so $f_*(\pi_1(Y, y_0)) = p_*(f)$ b is a special case of the
p, and $f:(Y,y_0) \to (X, \mathbb{R})$
onnected, then there is a
 $\pi_1(Y,y_0)) \subseteq p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$
 $\widetilde{\pi}_*(\pi_1(Y,y_0))) \subseteq p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$ \mathbf{r}_0
 \mathbf{r}_1
 \widetilde{x} then $f_* = p_* \circ f_*,$ so $f_*(\pi_1(Y, y_0)) = p_*(f_*(\pi_1(Y, y_0))) \subseteq p_*(\pi_1(X, \tilde{x}_0)).$ criterion: If $p:(X, x_0) \to (X, x_0)$ is a d
map, where Y is path-connected and loc
lift $\widetilde{f}:(Y, y_0) \to (\widetilde{X}, \widetilde{x}_0)$ of f (i.e., $f = p$
If \widetilde{f} exists, then $f_* = p_* \circ \widetilde{f}_*,$ so $f_*(\pi_1(Y, Y_0)) \subseteq p_*(\pi_1(\widetilde{X}, \Pi)$
 $\begin{bmatrix} \text{col} \ \text{a} \ \text{o} \ \text{y} \ \widetilde{x} \end{bmatrix}$ (0)), we will use path lifting to build the lift. Given $y \in Y$, choose a path γ in Y from y_0 to y and lift the path $f \circ \gamma$ in X
to a path f $\widehat{f}(\gamma)(x) = \widetilde{x}$ Then define $\widetilde{f}(y) = \widehat{f}(\gamma)(1)$ If well defined to a path $f \circ \gamma$ with $f \circ \gamma(0) = \tilde{x}_0$. Then define $f(y) = f \circ \gamma(1)$. If well-defined $(\circ \widetilde{f}_*, \text{ so } f_*,\ \circ \widetilde{f}_*, \text{ so } f_*,\ y_0))\subseteq p_*(\overline{g})\ \text{see a path}\ \circ \gamma(0)=\widetilde{x}$ and continuous, this is our required lift, since $(p \circ f)(y) = p(f(y)) = p(f \circ \gamma(1)) =$ $\widetilde{f}_*(\pi_1(Y, y_0))$ _{bu} $p \circ f \circ \gamma(1) = (f \circ \gamma)(1) = f(\gamma(1)) = f(y)$. For well-defined, if η is a path from y_0 to y_1 then $\alpha * \overline{n}$ is a loop so $f \circ (\alpha * \overline{n}) = (f \circ \alpha) * (f \circ \overline{n})$ is a loop giving an element of y, then $\gamma * \overline{\eta}$ is a loop, so $f \circ (\gamma * \overline{\eta}) = (f \circ \gamma) * (f \circ \eta)$ is a loop, giving an element of
 $f(\pi (V, \nu)) \subset \pi (\pi (\widetilde{V}, \widetilde{\pi}))$ and so lifts to a loop based at $\widetilde{\pi}$. So forward for lift to a path $\widetilde{f \circ \gamma}$ with $\widetilde{f \circ \gamma}(0) = \widetilde{x}_0$. Then define $\widetilde{f}(y) = \widehat{f}$
and continuous, this is our required lift, since $(p \circ \widetilde{f})(y) = p$
 $p \circ \widetilde{f \circ \gamma}(1) = (f \circ \gamma)(1) = f(\gamma(1)) = f(y)$. For well-defined, if
y, then $\begin{bmatrix} \gamma \\ \gamma \\ \vdots \\ \tilde{x} \end{bmatrix}$ 0. So $f \circ \gamma$ and $f \circ \eta$ lift, and continuous, this is our required lift, since $(p \circ \widetilde{f} \circ \gamma(1) = (f \circ \gamma)(1) = f(\gamma(1)) = f(y)$. For well y , then $\gamma * \overline{\eta}$ is a loop, so $f \circ (\gamma * \overline{\eta}) = (f \circ \gamma) * (\overline{f} \circ f_*(\pi_1(Y, y_0))) \subseteq p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$, and so lifts to 0, to have the same value at 1. So f is well-defined. Continuity comes
nly covered property of n Given $u \in V$ and a nbhd \widetilde{U} of $\widetilde{f}(u)$ in \widetilde{X} we $p \circ \overline{f} \circ \gamma(1) = (f \circ \gamma)(1) = f(\gamma(1)) = f(y)$. For well-defined, if η is a path from y
y, then $\gamma * \overline{\eta}$ is a loop, so $f \circ (\gamma * \overline{\eta}) = (f \circ \gamma) * (\overline{f} \circ \eta)$ is a loop, giving an element
 $f_*(\pi_1(Y, y_0)) \subseteq p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$ \mathcal{U} of $f(y)$ in X, we want a nbhd V of y with $f(V) \subseteq U$. Choose an evenly covered nbhd \mathcal{U}_y for $f(y)$, the choose $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{U}}$ curve $\widetilde{\mathcal{$ $f \circ (\gamma * \overline{\eta})$
 $(\gamma * \overline{\eta})$, and
 $\overline{\alpha}$ and $\overline{\beta}$ and $\overline{\beta}$
 $\widetilde{f}(\mathcal{V}) \subseteq \mathcal{U}$ $f_*(\pi_1(Y, y_0)) \subseteq p_*(\pi_1(\tilde{X}, \tilde{x}_0)),$ and so lifts to a loop based a
starting at \tilde{x}_0 , to have the same value at 1. So \tilde{f} is well-
from the evenly covered property of p. Given $y \in Y$, and
want a nbhd V of y wi y over \mathcal{U}_y which contains $f(y)$, and set $\mathcal{W} = \mathcal{U} \cap \mathcal{U}_y$. p is a homeo from W to
on set $p(\mathcal{W}) \subset X$. Then if we set $\mathcal{V}' = f^{-1}(p(\mathcal{W}))$ this is open and contains u $\frac{1}{4}$ the open set $p(\mathcal{W}) \subseteq X$. Then if we set $\mathcal{V}' = f^{-1}(p(\mathcal{W}))$ this is open and contains y,
and so contains a path-connected open phhd V of u. Then for every point $z \in \mathcal{V}$ we and so contains a path-connected open nbhd $\mathcal V$ of y. Then for every point $z \in \mathcal V$ we compute $f(z)$ by a path γ from y_0 to z which first goes to y and then, *in* V, from y
to z. Then by unique path lifting since $f(3) \subset \mathcal{U}$ f ox lifts to the concatenation -
-
to z. Then by unique path lifting, since $f(V) \subseteq U_y$, $f \circ \gamma$ lifts to the concatenation
of a path from $\widetilde{f}(v)$ and a path in \widetilde{U} from $\widetilde{f}(v)$ to $\widetilde{f}(v) \subseteq \widetilde{U}$ sheet \mathcal{U}_y over \mathcal{U}_y which contains $f(y)$, and set $W = \mathcal{U} \cap \mathcal{U}_y$. p is a homeo in
the open set $p(\mathcal{W}) \subseteq X$. Then if we set $\mathcal{V}' = f^{-1}(p(\mathcal{W}))$ this is open and co
and so contains a path-connected open o to $f(y)$ and a path in \mathcal{U}_y from $f(y)$ to $f(z)$. So $f(z) \in \mathcal{U}$.