Math 872 Problem Set 6

Starred (*) problems are due Thursday, April 2.

40. Describe how to express the Cartesian product of two simplices $\Delta^n = [x_0, \ldots, x_n]$ and $\Delta^m = [y_0, \ldots, y_m]$ as a Δ-complex, so that each $\Delta^n \times \{y_i\}$ and $\{x_j\} \times \Delta^m$ are simplices.

(*) 41. [Hatcher, p.131, # 1] What familiar space is the quotient Δ-complex of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the order of vertices?

42. Suppose that X is a path-connected space, equipped with a Δcomplex structure. Show, directly from the definitions, that $H_0^\Delta(X) \cong \mathbb{Z}$. (You may assume without proof that the one-skeleton is connected.)

43. Compute the simplicial homology groups of the 3-simplex Δ^3 (that is, the Δ-complex obtained from 4 vertices by gluing on 6 1-simplices, 4 2-simplices and a single 3-simplex in the "obvious" way).

(*) 44. [Hatcher, p.131, # 7] Find a way of identifying the faces of a 3-simplex Δ^3 to produce a Δ-complex structure on S^3 having a single 3-simplex, and compute the simplicial homology groups of this Δ-complex.

45. Compute the simplicial homology groups of the two-sphere, directly from the definitions, using the $Delta$complex structure coming from the boundary of a tetrahedron (i.e., 3-simplex).

(*) 46. In the group $G = \mathbb{Z} \times \mathbb{Z}$, consider the subgroup H generated by $(-5, 1)$ and $(1, -5)$. Show that the quotient group G/H is cyclic. Which of the standard cyclic groups is it isomorphic to?

47. Suppose that A and B are subgroups of the abelian group C. We define $A + B = \{a + b \ : \ a \in A, b \in B\} \subseteq C$.

    (1) Show that $A + B$ is a subgroup of C.

    (2) There is a natural homomorphism $\phi : A \times B \to A + B$, defined by $\phi(a, b) = a + b$. Show that ϕ is surjective, and show that ϕ is an isomorphism if and only if $A \cap B = \{0\}$.