Math 872 Problem Set 7

Starred (*) problems are due Thursday, April 9.

48. [Hatcher, p.131, #9] Compute the simplicial homology groups of the Δ-complex X obtained from the n-simplex $\Delta^n = [v_0, \ldots v_n]$ by identifying every face of the same dimension to one another (respecting the implied orientations from the ordering of the vertices, so, e.g, $[v_1, v_2, v_4]$ is glued to $[v_0, v_3, v_4]$ via the homeomorphism that sends v_0 to v_1, v_2 to v_3, etc.). Thus X has a single k-simplex for each $k \leq n$.

49. Show that the Smith normal form of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$. Explain why this implies that $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \cong \mathbb{Z}_6$.

(*) 50. Find the Smith normal form of the matrix $\begin{pmatrix} 3 & 5 & 9 \\ 1 & -1 & 1 \\ 5 & 5 & 11 \end{pmatrix}$

50. Find the Smith normal form of the matrix $\begin{pmatrix} 2 & 2 & 5 \\ 2 & 3 & 1 \\ 5 & 2 & 6 \end{pmatrix}$

(*) 51. Regarding the n-simplex $X = \Delta^n$ as a Δ-complex in the natural way, show that if $A \subseteq X$ is a subcomplex with $H_{n-1}(A) \neq 0$, then $A = \partial\Delta^n$. (Hint: show that an $(n-1)$-cycle for A (and hence for X) must either be 0 or have non-zero coefficient for every $(n-1)$-dimensional face of X.)

(*) 52. [Hatcher, p.132, #11] Show that if $A \subseteq X$ is a retract of X, then the inclusion map $\iota : A \to X$ induces an injection on all singular homology groups.