Math 872 Problem Set 5

Starred (*) problems are due Thursday, March 11.

32. [Hatcher, p.19, #16] Show that the CW-complex $S^\infty = \bigcup_n S^n$ is contractible.

    [Hint: On $S^\infty \times [1/n, 1/(n-1)]$, assume you have already mapped into $S^{n-1} \subseteq S^\infty$, and deform it (through S^n) into (a hemisphere of) S^{n-2} .]

(*). 33. [Hatcher, p.19, #9] Show that if X is a contractible space, and there is a retraction $r : X \to A$ from X to a subspace A of X, then A is contractible. [Note: your goal is to build a homotopy $H : A \times I \to A$ from a constant map to the identity map.]

34. Show that if X is a space and $f, g : \partial D^n \to X$ are attaching maps of an n-cell, giving $Y_f = X \cup_f D^n$ and $Y_g = X \cup_g D^n$, then $f \simeq g$ (via a homotopy H) implies that $Y_f \simeq Y_g$. [Hint: Build $Y_H = X \cup_H (D^n \times I)$! And find some deformation retractions...]

    [So homotopic attaching maps give homotopy equivalent spaces! This can be useful in doing π_1 (and other) calculations.]

(*) 35. [Hatcher, p.79, # 1] Show that if $p : \widetilde{X} \to X$ is a covering map and $A \subseteq X$ is a subspace if X, then

    $p|_{p^{-1}(A)} : p^{-1}(A) \to A$ is also a covering map.

36. [Hatcher, p.79, # 2] Show that if $p_1 : \widetilde{X_1} \to X_1$ and $p_2 : \widetilde{X_2} \to X_2$ are covering maps, then the map

    $p = p_1 \times p_2 : \widetilde{X_1} \times \widetilde{X_2} \to X_1 \times X_2$, given by $p(x_1, x_2) = (p_1(x_1), p_2(x_2))$, is also a covering map.

37. [Hatcher, p80, # 11] Find a pair of (finite) graphs (= 1-dim'l CW complexes with finitely many 0- and 1-cells) X_1 and X_2 that have a common finite-sheeted covering space $p_1 : X \to X_1$, $p_2 : X \to X_2$, but do *not* commonly cover another space, i.e., they are not both covering spaces of a single space Y.

(*) 38. Show that if $p : X \to Y$ and $q : Y \to Z$ are both covering maps, and, for each $z \in Z$, $q^{-1}(z)$ is *finite*, then $q \circ p : X \to Z$ is also a covering map.

    [N.B. This result can be false, if $q^{-1}(z)$ is infinite.]

39. Suppose that $p : \widetilde{X} \to X$ and $q : \widetilde{Y} \to Y$ are finite-sheeted covering spaces, with all spaces connected. Describe how to use these to build connected finite-sheeted covering spaces of the one-point union $X \vee Y$. What is the smallest number of sheets that such a covering can have?