Math 872 Problem Set 8

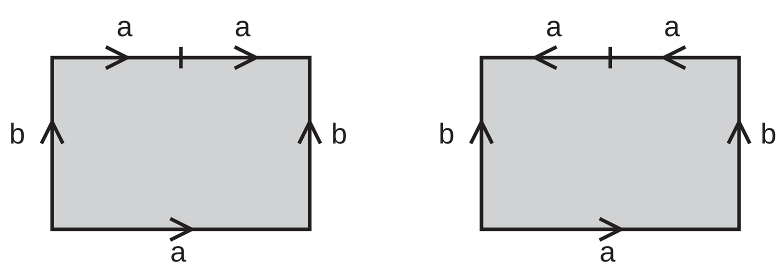
Starred (*) problems are due Thursday, April 14.

56. [Hatcher, p.131, # 1] What familiar space is the quotient Δ-complex of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the order of vertices?

(**) 57. Compute the simplicial homology groups of the two-sphere, directly from the definitions, using the Δ–complex structure coming from the boundary of a tetrahedron (i.e., 3-simplex).

58. [Hatcher, p.131, #9] Compute the simplicial homology groups of the Δ-complex X obtained from the n-simplex $\Delta^n = [v_0, \dots v_n]$ by identifying every face of the same dimension to one another (respecting the implied orientations from the ordering of the vertices, so, e.g, $[v_1, v_2, v_4]$ is glued to $[v_0, v_3, v_4]$ via the homeomorphism that sends v_0 to v_1, v_2 to v_3, etc.). Thus X has a single k-simplex for each $k \leq n$.

59. Find a Δ-complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $S^1 \times \{1\}$ to $S^1 \times \{0\}$ by a map representing 2 times the generator of $\pi_1(S^1)$. (See figure below, left.)



(**) 60. Find a Δ-complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $S^1 \times \{1\}$ to $S^1 \times \{0\}$ by a map representing -2 times the generator of $\pi_1(S^1)$. (See figure above, right!)

61. Regarding the n-simplex $X = \Delta^n$ as a Δ-complex in the natural way, show that if $A \subseteq X$ is a subcomplex with $H_{n-1}(A) \neq 0$, then $A = \partial\Delta^n$.

[Hint: show that an $(n-1)$-cycle for A (and hence for X) must either be 0 or have non-zero coefficient for <u>every</u> $(n-1)$-dimensional face of X.]

62. Show that the Smith normal form of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$. Explain why this implies that $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \cong \mathbb{Z}_6$.

(**) 63. Find the Smith normal form of the matrix $\begin{pmatrix} 3 & 5 & 9 \\ 1 & -1 & 1 \\ 5 & 5 & 11 \end{pmatrix}$.

[Hint/note: the determinant can give a rough check that your SNF 'might' be correct...]

64. Find the Smith normal form of the matrix $\begin{pmatrix} 2 & 2 & 5 \\ 2 & 3 & 1 \\ 5 & 2 & 6 \end{pmatrix}$.