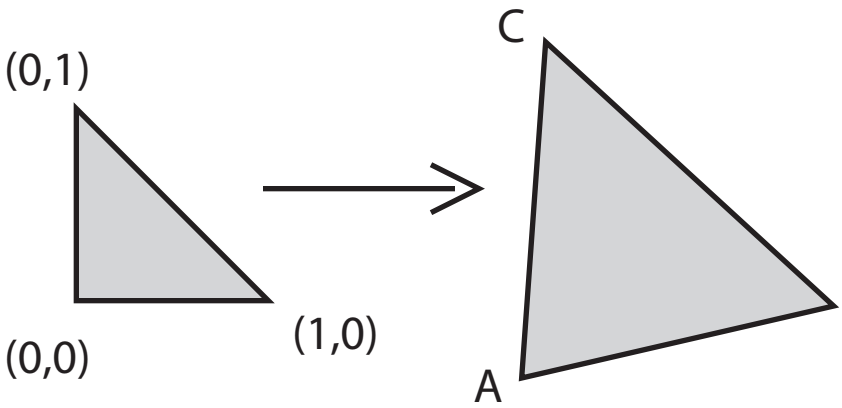
Building homotopies the way you want to (in pieces):

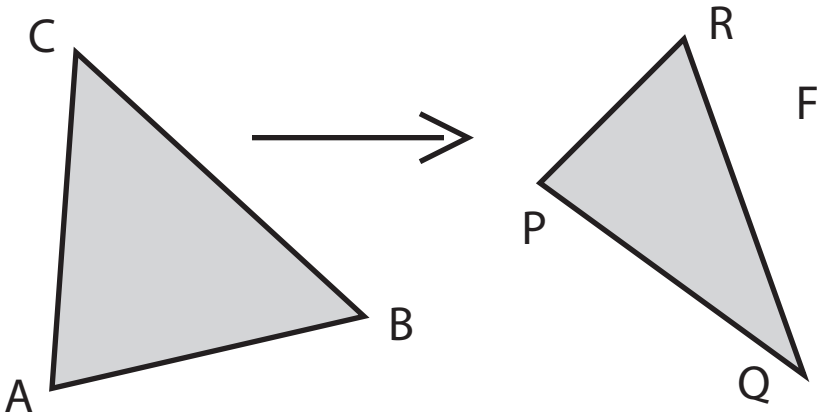


$T(x,y)=A+x(C-A)+y(B-A)$

T is a cts bijection between the triangles, hence a homeo.

Inverse is $S(z)=(a(z),b(z))$

(Think: parametrization of the triangle)

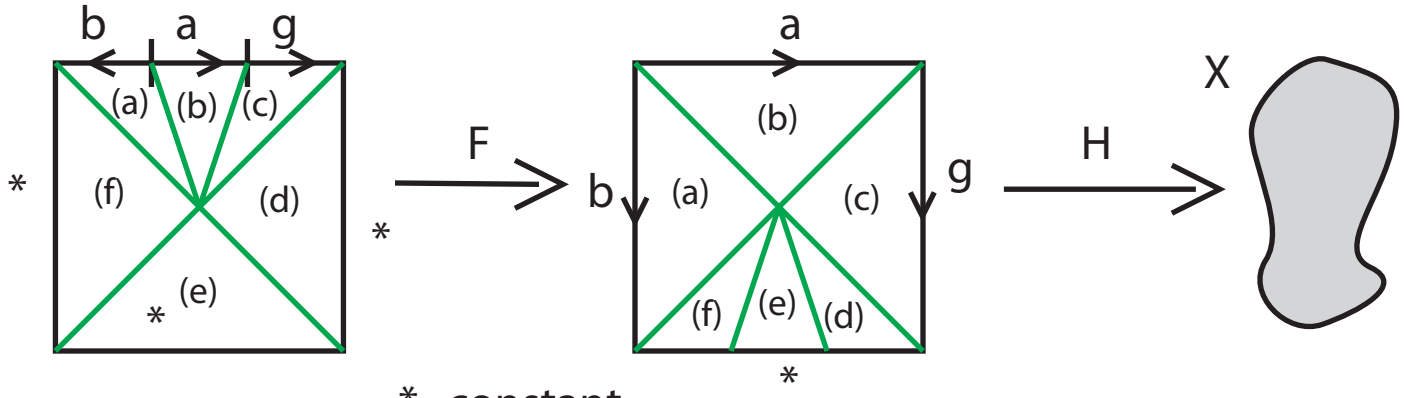


$F(z)=P+a(z)(Q-P)+b(z)(R-P)$

is then a cts map between the triangles, sending A to P, B to Q, and C to R, and linear on the edges.

Together with the pasting lemma, this allows us to build useful maps by cutting domain and codomain into triangles (usually with the same combinatorial pattern), and then using the above maps between them.

E.g.,



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