Math 872 Topology II

Exam 1

Instructions: Your solutions are due Thursday, April 1, by the end of the day. In preparing your answers, you may consult our textbooks, your course notes, your solutions to the problem sets, the instructor's solutions to the problem sets, and the 'board notes' prepared by the instructor. You should not consult any other resource (to the extent that you other studies allow) to aid you in formulating your solutions, nor discuss the exam problems with anyone other than your instructor, except on trivial matters, until after the official exam time ends. Should you have any questions on the meaning or scope of the questions, or any other questions about the problems and their solutions, feel free to discuss them with your instructor at any time that he is available.

Each problem (number) has equal weight.

Exam 1 Problems:

1. The isomorphism $(\mathrm{pr}_X)_* \times (\mathrm{pr}_Y)_* : \pi_1(X \times Y, (x_0, y_0)) \to \pi_1(X, x_0) \times \pi_1(Y, y_0)$ implies that for loops γ mapping into $X \times \{y_0\}$ and η mapping into $\{x_0\} \times Y$, we must have $\gamma * \eta \simeq \eta * \gamma$ rel endponts (i.e., the two elements $[\gamma]$ and $[\eta]$ of $\pi_1(X \times Y, (x_0, y_0))$ commute). Build an explicit homotopy $H : I \times I \to X \times Y$ between the two products.

   [Note/hint: You are 'really' building two maps, one into X and one into Y. What do $\mathrm{pr}_X \circ (\gamma * \eta)$, etc. look like?]

2. Let X be the quotient space formed from the 2-sphere S^2 by identifying the north and south poles, $(0, 0, 1)$ and $(0, 0, -1)$. Put a cell structure on X to make it a CW-complex and use this to compute (a presentation for) $\pi_1(X)$.

3. If X and Y are connected, locally path connected spaces and $p : (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)$ and $q : (\widetilde{Y}, \widetilde{y}_0) \to (Y, y_0)$ are connected coverings, explain when a continuous map $f : (X, x_0) \to (Y, y_0)$ <u>lifts</u> to a continuous map $\widetilde{f} : (\widetilde{X}, \widetilde{x}_0) \to (\widetilde{Y}, \widetilde{y}_0)$, with $q \circ \widetilde{f} = f \circ p$. Is the lifted map $\widetilde{f}$ always unique?

4. In class we showed (I think?) that $G = \langle a, b \mid a^2, b^3, abab \rangle$ is a presentation for the symmetric group on 3 letters, S_3 (by mapping $a \mapsto (12)$ and $b \mapsto (123)$, although this is relatively unimportant to the rest of the problem...). The covering space of $X = S^1 \vee S^1$ corresponding to the kernel of the homomorphism $\varphi : F(a, b) = \langle a, b \mid \ \rangle \to G$ sending a to a and b to b then has the property that every conjugate of the words $a^2, b^3, abab$ lifts to a loop. Build this covering space (and corresponding covering map).

   [Note: we <u>know</u> what degree this covering should have...]