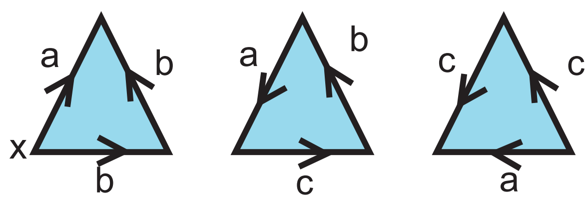
Math 872 Exam 2

Your solutions should be uploaded to the class Canvas page (under the "Exam 2" link) by the end of the day on Tuesday, May 4. In formulating your solutions, you may consult our textbooks, your course notes, your solutions to the problem sets, the instructor's solutions to the problem sets, and the 'board notes' prepared by the instructor. You should not consult any other resource (to the extent that you other studies allow) to aid you in formulating your solutions, nor discuss the exam problems with anyone other than your instructor, except on trivial matters, until after the official exam time ends. Should you have any questions on the meaning or scope of the questions, or any other questions about the problems and their solutions, feel free to discuss them with your instructor at any time that he is available. **All problem numbers have equal weight.**

1. Let $p : \widetilde{X} \to X$ be a **finite-sheeted** covering map. Show that if X is compact and Hausdorff, then $\widetilde{X}$ is compact and Hausdorff.

2. Find the (simplicial) homology groups of the Δ-complex X obtained by gluing three 2-simplices together in the pattern shown below (all edges marked 'a' are identified in the indicated orientation, etc.).



[You'll probably want to first work out how many vertices the Δ-complex has...]

3. We've seen that a covering map $p : \widetilde{X} \to X$ induces an injective map on fundamental groups. Show, however, that this need not be true for homology: find a covering map $p : \widetilde{X} \to X$ between path-connected spaces so that for at least one k we have $p_* : H_k(\widetilde{X}) \to H_k(X)$ is <u>not</u> injective. [Path-connected is 'just' so that you don't use {two points} $\to$ {one point} for your example; so your k will need to be bigger than 0...]

4. For a topological space X, the *cone on* X, cX, is the quotient space $(X \times [0,1])/(X \times \{0\})$ (i.e., the product of X with I, with a copy of X crushed to a point). The *suspension of* X, ΣX, is the quotient space $(X \times [0,2])/\sim$, where $(x,t) \sim (y,s)$ if $t = s = 0$ <u>or</u> $t = s = 2$.

(a) Show that cX is contractible (in fact, it deformation retracts to the 'cone point' $[X \times \{0\}]$), and the suspension on X can be expressed as $\Sigma X = A \cup B$, with $A \cong B \cong cX$ and $A \cap B \cong X$.

(b) Use a long exact homology sequence to find a formula expressing the singular homology groups $\widetilde{H}_k(\Sigma X)$ of the suspension in terms of the homology groups of X. [You may take as given (or prove!) that your sets A and B in part (a) have neighborhoods in X that deformation retract to them.]