Course Offering - Fall 1999

Math 971 Algebraic Topology

Instructor: Mark Brittenham

Topology is the study of topological spaces (= sets in which a rough sense of points being 'close to' other points is given) and continuous functions (= functions preserving this 'closeness') between them. Algebra is the study of 'algebraic objects' (= sets having some kind of multiplication and/or addition, such as groups, rings, or fields) and homomorphisms (= functions preserving the multiplication and/or addition). With such broad similarities between them - both disciplines study sets with additional structure, and functions preserving this structure - it was inevitable that people would find ways to connect them. Algebraic topology is the practice of using algebra to understand topology. That is, we find ways to assign algebraic objects and functions to topological ones. This allows us to associate an (easier?) algebraic problem or setting to a topological one. A solution to the algebraic problem can then be used to shed light on the original topological problem.

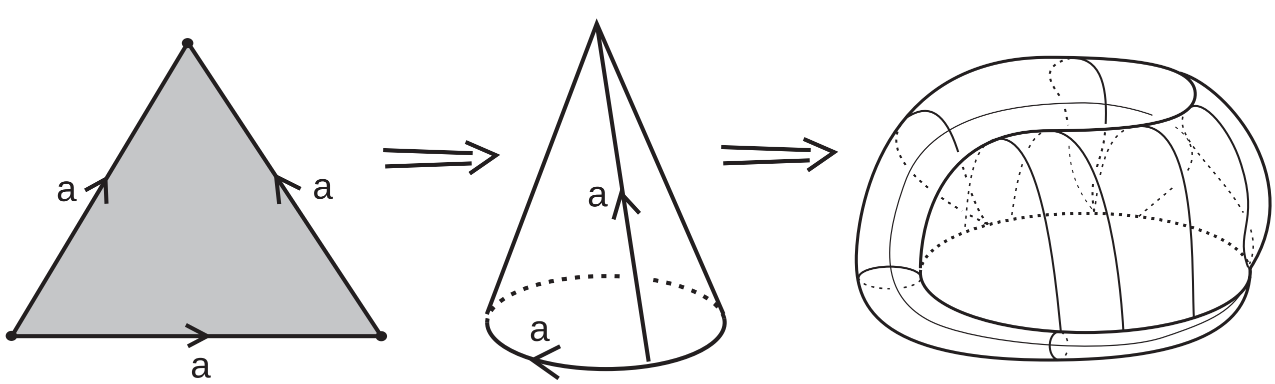
Example: There is no continuous map from the sphere to the circle which takes the equator of the sphere to the circle in a one-to-one fashion. Using one of the associations we will build in this course, this turns out to follow from the fact that the identity homomorphism from the integers to itself is not the same as the zero homomorphism! (This is certainly an easier (algebraic) property to prove...)

Another example: for any continuous function f from the sphere to plane, there is a point x in the sphere so that $f(x) = f(-x)$. Put another way, at every moment in time, there are always two antipodal points on the surface of the Earth where the temperature and pressure are *exactly* the same. (The associated algebraic problem is related to the integers mod 2.)

In recent years, this route from topology to algebra and back to topology has often been reversed. The field of geometric group theory, for example, is, in large part, about taking an algebraic setting, associating to it a topological setting, and using our understanding of the topology to get a better understanding of the algebra.

To learn more about this fascinating interplay between algebra and topology, and what it means to the rest of mathematics (and physics and chemistry and economics...), come and take this course! The roads we can lead ourselves down include recent work aimed at determining which topological space our universe really is (and it's probably *not* Euclidean space!). The connections to *all* of these far-flung disciplines is probably beyond the scope of a one-semester course , but we will certainly make a good start!

Oh, and I can't write an advertisement like this without throwing in a picture, so here is one - it's what you get when you glue all three sides of a triangle together.



The Topologists' "Dunce Hat"