Problems from Munkres assigned Fall 2004, of "qual" caliber:

p.118 # 10(a): Let A be a set, $\{X_\alpha\}_{\alpha\in J}$ a family of topological spaces, and $\{f_\alpha\}_{\alpha\in J}$ a family of functions $f_\alpha : A \to X_\alpha$. Show that there is a unique coarsest topology on A relative to which each of the f_α is continuous.

p.111 # 8: Let $f, g : X \to \mathbb{R}$ be continuous. Let $h : X \to \mathbb{R}$ be defined by $h(x) = \min\{f(x), g(x)\}$ for all $x \in X$. Show that h is continuous.

p.118 # 3: Show that the product of Hausdorff spaces is Hausdorff.

p.101 # 8: Let A_α be subsets of a topological space X. Determine whether $\overline{\cap A_\alpha} = \cap\overline{A_\alpha}$ and if not, determine whether either containment $\subset$ or $\supset$ holds.

p.152 # 11: Let $p : X \to Y$ be a quotient map such that $p^{-1}(\{y\})$ is connected for each $y \in Y$. Show that if Y is connected, then so is X.

p.158 # 8(b): If $A \subset X$ and A is path connected, is $\overline{A}$ necessarily path connected?

p.171 # 5: Show that if A, B are disjoint compact subsets of the Hausdorff space X, then there are disjoint open sets $\mathcal{U}, \mathcal{V} \subseteq X$ with $A \subseteq \mathcal{U}$, $B \subseteq \mathcal{V}$.

p.199 # 5: Let Y be a Hausdorff space and let $f, g : X \to Y$ be continuous. Show that $\{x \mid f(x) = g(x)\}$ is closed in X.

p.218 # 3: Let X be a compact Hausdorff space. Show that X is metrizable iff X has a countable basis.

p.330 # 3(b): Show that a contractible space is path connected.

p.335 # 4: Let $a_0 \in A \subset X$ and let $r : X \to A$ be a retracion. Show that $r_* : \pi_1(X, a_0) \to \pi_1(A, a_0)$ is surjective.

p.341 # 6(a): Let $p : E \to B$ be a covering map. Show that if B is Hausdorff, then so is E.

p.353 # 1: Show that if A is a retract of B^2, then every continuous map $f : A \to A$ has a fixed point.

Problems from Hatcher so far assigned, of "qual" caliber:

p.39 # 13 : If $x_0 \subseteq A \subseteq X$ with A path-connected, then the inclusion-induced map $\pi_1(A, x_0) \to \pi_1(X, x_0)$ is surjective $\Leftrightarrow$ every path in X with endpoints in A is homotopic, rel endpoints, to a path in A.

p.39, # 14 : Show that $\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y)$

p.40, # 20 : If $f_t : X \to X$ is a homotopy from the identity map to itself, then $t \mapsto f_t(x_0)$ is an element in the center of $\pi_1(X, x_0)$.

p.53, # 6 : Attaching an n-cell, $n \geq 3$ to a space does not change its π_1 .

p.53, # 7: Compute the fundamental group of the space obtained from S^2 by identifying the north and south poles.

p.53, # 8: Compute the fundmental group of the space obtained by gluing the curves $S^1 \times \{x_0\}$ in two copies of a 2-torus together.

p.54, # 14 : Compute the fundamental group of the space obtained from the cube by identifying opposite faces, each with a 90-degree right-hand twist.

p.79, # 1 : If $p : \widetilde{X} \to X$ is a covering map, and $A \subseteq X$, then $p : p^{-1}(A) \to A$ is also a covering map.

p.79, # 3: If $p : \widetilde{X} \to X$ is a finite-sheeted covering, then $\widetilde{X}$ is compact and Hausdorff $\Leftrightarrow$ X is.

p.79, # 9: If X is path-connected and locally path-connected, with $\pi_1(X)$ finite, then every map $f : X \to S^1$ is null-homotopic.

p.80, # 15: If $p : \widetilde{X} \to X$ is a universal covering, $A \subseteq X$ path-connected and locally path-connected, and $\widetilde{A}$ a path-component of $p^{-1}(A)$, then $p : \widetilde{A} \to A$ is the covering space corresponding to the kernel of the map $i_* : \pi_1(A) \to \pi_1(X)$.

p.86, # 2: If $A \subseteq X$ are connected graphs, then there is a retraction $r : X \to A$.
[Maybe it's enough to do this for finite graphs?]