UNL Mathematics Sample Qualifying Exam II **Math 970/971**

Do three of the problems from section A and three questions from section B. If you work more than the required number of problems, make sure that you clearly mark which problems you want to have counted. If you have doubts about the wording of a problem or about what results may be assumed without proof, please ask for clarification. In no case should you interpret a problem in such a way that it becomes trivial.

Section A: Point Set Topology.

1. Let $\{A_\alpha\}$ be a collection of subsets of a topological space X. Determine whether $\overline{\cup A_\alpha} = \cup \overline{A_\alpha}$ holds; if equality fails, determine whether one of the inclusions $\subset$ or $\supset$ holds.

2. Let $(X, \mathcal{T})$ be a compact Hausdorff topological space, and let $\mathcal{T}'$ be another topology on X with $\mathcal{T}' \subset \mathcal{T}$ and $\mathcal{T}' \neq \mathcal{T}$. Show that the space $(X, \mathcal{T}')$ is compact but *not* Hausdorff.

3. A topological space X is called *completely Hausdorff* if for every pair a, b of distinct points in X, there are open sets U, V in X satisfying $a \in U$, $b \in V$, and $\overline{U} \cap \overline{V} = \emptyset$. Show that a product of two completely Hausdorff spaces is completely Hausdorff.

4. Show that a connected, locally path-connected, space is path-connected.

Section B: Homotopy and Homology.

5. Let L_1, L_2 be disjoint lines in $\mathbb{R}^3$, and $X = \mathbb{R}^3 \setminus (L_1 \cup L_2)$. Compute $\pi_1(X)$.

6. Show that every continuous map $f : \mathbb{R}P^2 \to S^1$ is homotopic to a constant map.

7. Find a Δ-complex structure on the space X obtained by identifying a point a in the 2-sphere with a point b in the 2-torus, and compute the simplicial homology groups of X.

8. Prove that if $\mathbb{R}^n$ is homeomorphic to $\mathbb{R}^m$, then $n = m$.