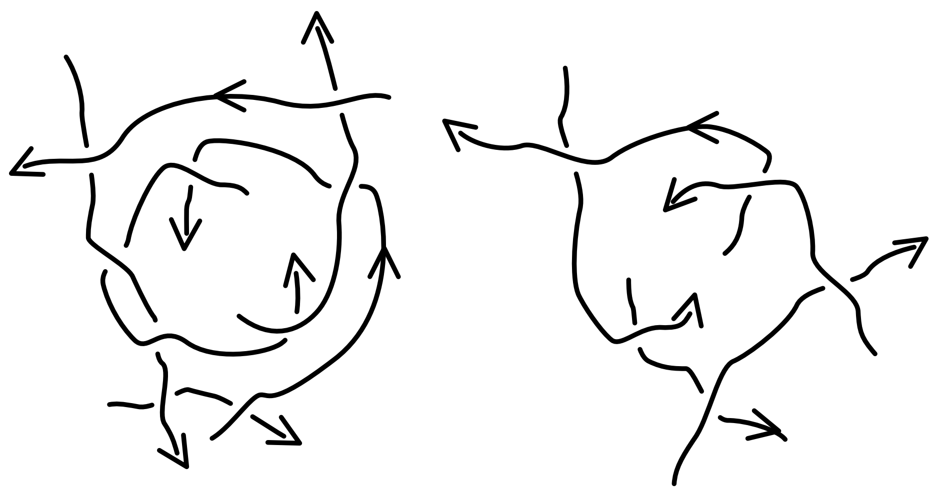
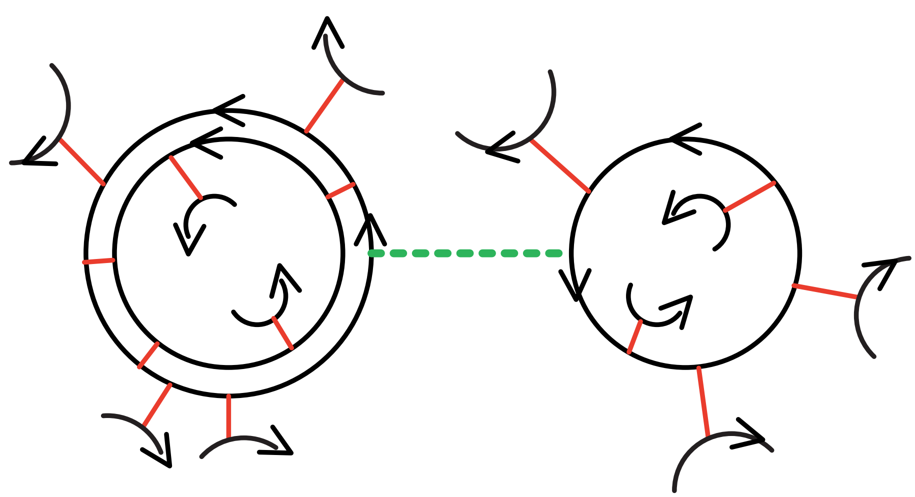
Yamada: turning a projection for a knot/link into a braid, without changing the number of Seifert circles:

The idea: if the Seifert circles are all parallel, forming a "bullseye", then the center of the bullseye is an axis around which the knot/link runs, making the knot/link a braid. We want to find a projection having this property, without changing the number of Seifert circles for the projection.

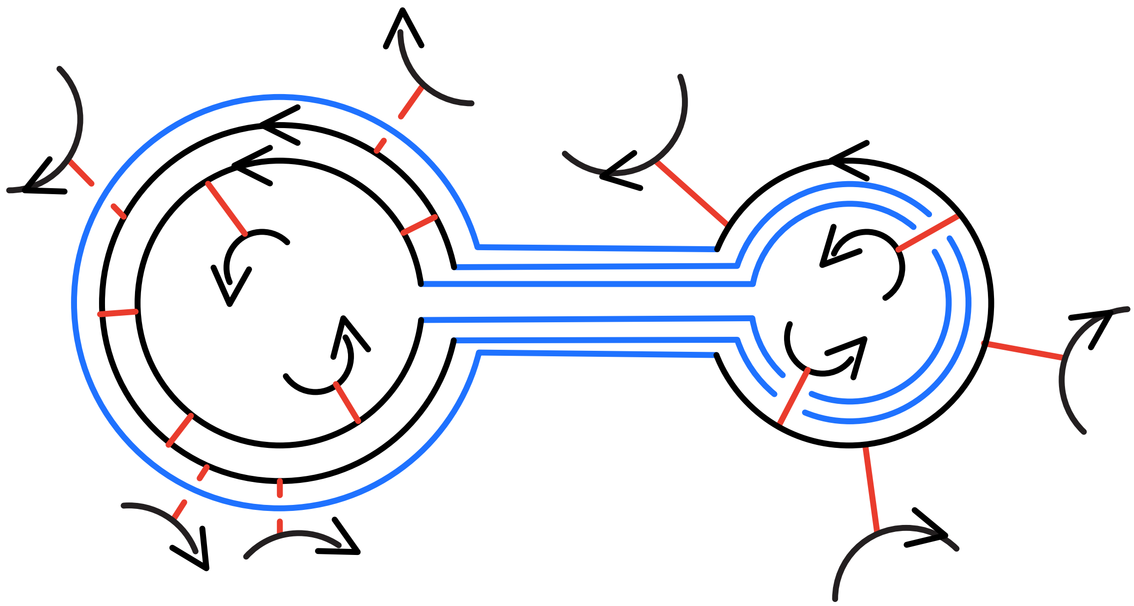
Starting with the knot/link, find the Seifert circles. (Red segments represent the twisted bands.) The goal: find a new projection with fewer distinct families of parallel circles among the Seifert circles.

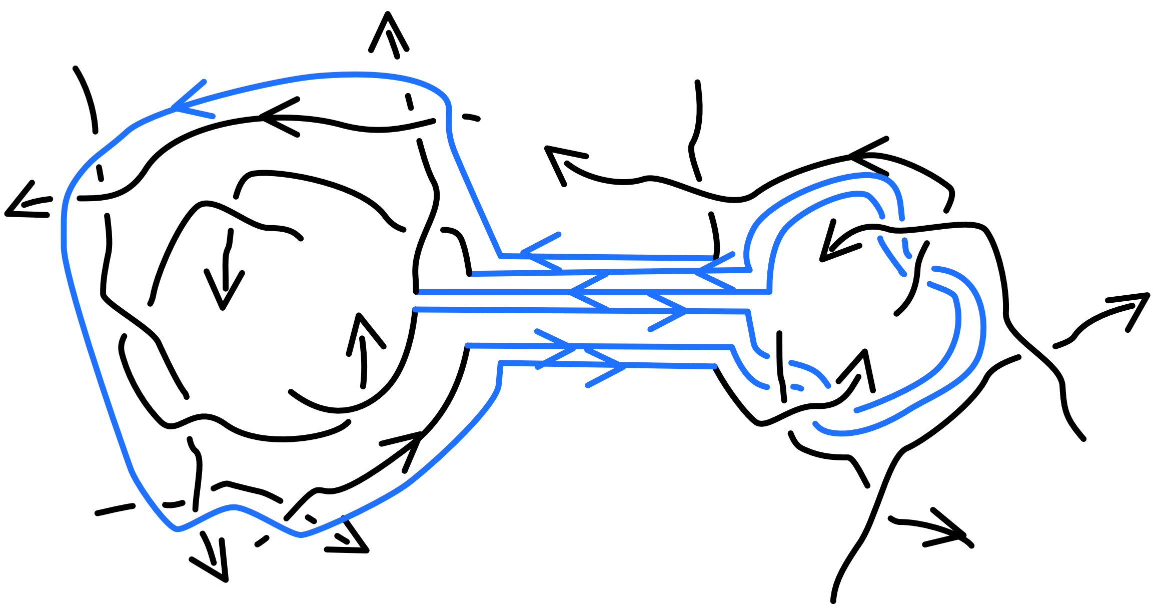




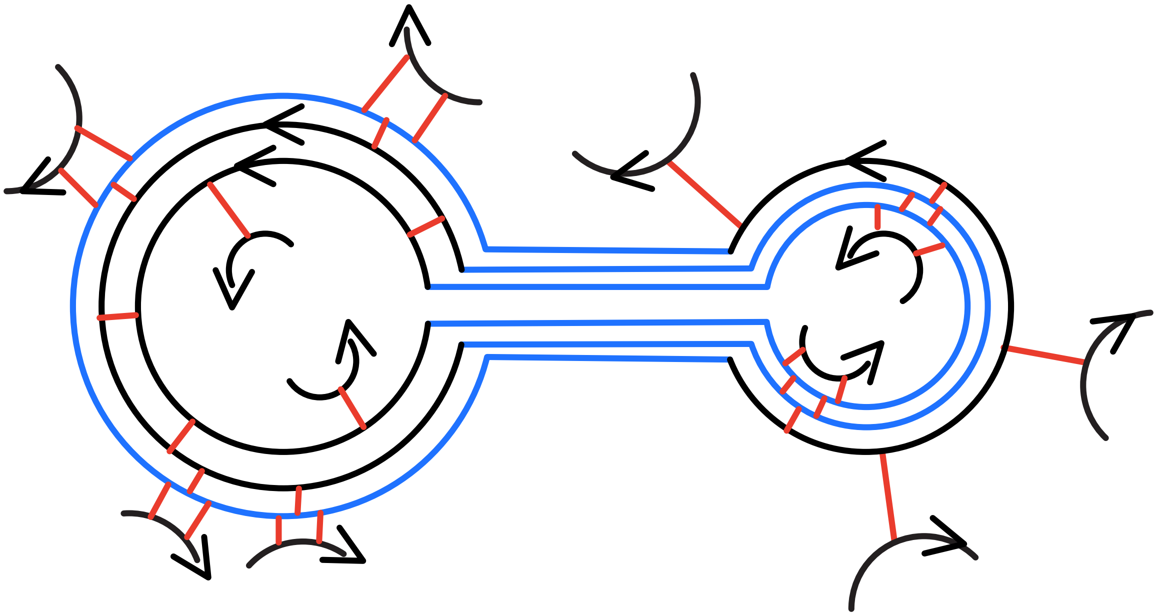
The idea: find a an arc, missing the circles and the segments, which joins together two different families of parallel circles having the same (clockwise or counter-clockwise) orientation.

Use this arc to extend the two sets of Seifert disks one over/under one another, to make their boundaries become a single set of parallel circles.





Problem: our original surface is **not** the one that Seifert's algorithm would build for the resulting new projection! [The new circles pass over/under some of our old twisted bands.] But if we run Seifert's algorithm on the new projection, we **do** get the same Seifert circles! We just have (a lot) more twisted bands. Basically, this is because the added blue arcs are oriented in the same way as the boundaries of the Seifert disks they are running parallel to.



Finding the arc to amalgamate disks along: if the Seifert circles are **not** all parallel, then there is a complementary region of the Seifert circles which is neither a disk nor an annulus (i.e., we see at least three distinct Seifert circles while standing in the middle). At most one of the associated Seifert disks D may contain all of the others (i.e., all others are nested within it). If we choose the opposite disk (on the projection 2-sphere) for the circle corresponding to D, then none of the disks are nested. [This switch of Seifert disk can be thought of as choosing a different point at infinity for the 2-sphere, to put the projection in the plane, which does not affect the Seifert circles.] When the disks are not nested, a twisted band between them implies that the orientations of their boundaries are opposite. Since we see at least three disks, two of them must have the same orientation; we choose these to join by our green arc.